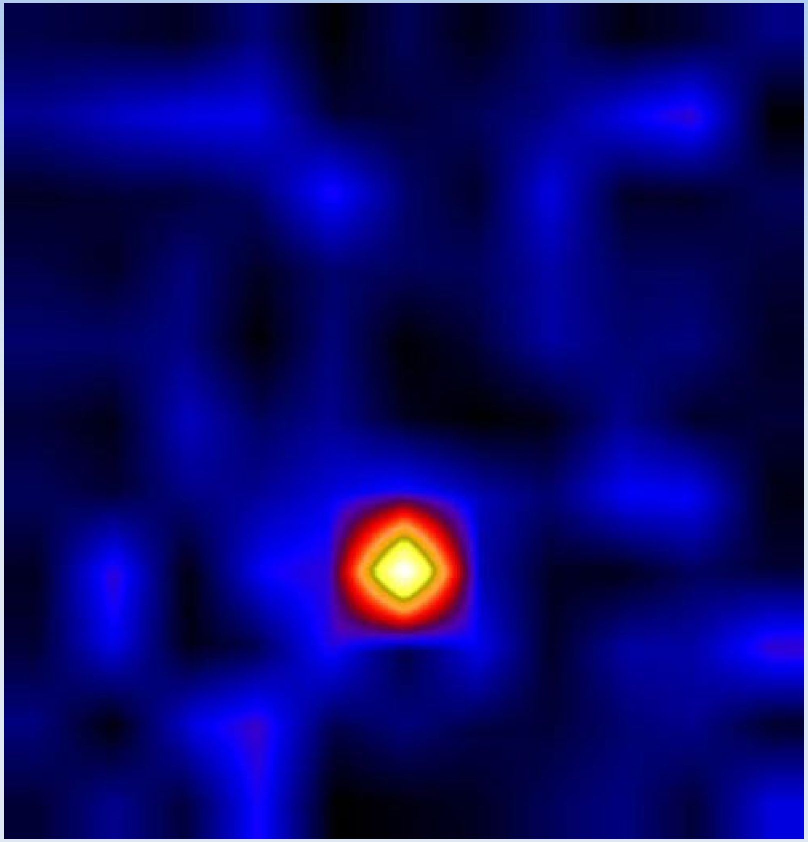
Compact Stars



Lecture 3

Summary of the previous lecture

- I presented stellar structure equations, and evolution of Solar-mass star on the HR diagram
- I talked about close binary systems
  - Interacting binaries, their classification (detached, semi-detached or contact systems)
  - Definition of Roche lobe, position of Largrange points, fitting formulae for different mass ratios of stars
  - The process of Roche lobe overflow, which leads to the formation of accretion disks
- I presented classification of binaries wrt. type of donor and accretor (compact star, evolved star, MS star) – mostly phenomenological, based on known archetypical systems

Today I will derive the basic properties of accretion disks in binaries.

Reminder: Virial theorem

- In statistical mechanics, the virial theorem provides a general equation that relates the time-average of the total kinetic energy of a stable system of discrete particles, bound by a conservative force, with that of the total potential energy of the system.

$$\langle K \rangle = -1/2 \, \Sigma_{k=1}^{N} \langle F_k \, r_k \rangle$$

- If the force between any two particles of the system results from a potential energy $U(r) = \alpha r^n$, the virial theorem takes the simple form

$$2 \langle K \rangle = n \langle U_{TOT} \rangle .$$

- U_{TOT} represents the total potential energy of the system, i.e., the sum of the potential energy U(r) over all pairs of particles. A common example of such a system is a star held together by its own gravity, where $n = -1$.

Reminder: Opacity

- Opacity of a shell in the star is found by adding together the cross-sections of all the absorbers and scatterers in the shell, and dividing by the total mass of the shell.

$$\kappa = \frac{\sum_i n_i \sigma_i}{\rho}$$

- Simplest opacity is given by Thomson scattering, when the photons are scattered over free electrons. The cross-section of this process is

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2$$

which gives 6.7×10^{-25} cm^2

Circularization radius

- Circular orbit: Keplerian velocity at radius R

$$V_K = \sqrt{\frac{GM}{R}}$$

- Angular momentum is conserved

$$R_c V(R_c) = \frac{2\pi}{P_{orb}} R_{L1}^2$$

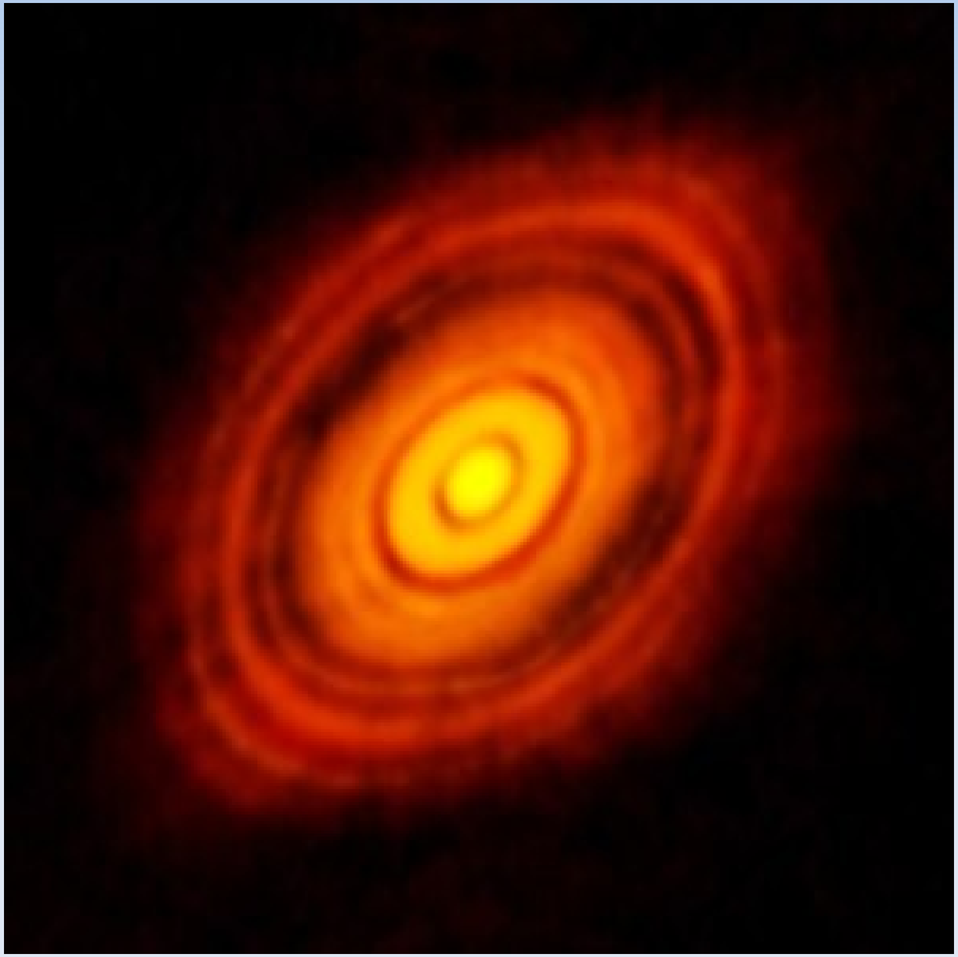
- Final formula $R_c/a = (1+q)(R_{L1}/a)^4$
- Approximately: $R_c/a = 0.0859\, q^{-0.426}$
- Various fitting fomulae, some give R_{L2}: q-> q^{-1}

Accretion

Accretion is astrophysical process of accumulating gaseous matter onto a massive object by gravitational attraction.

Most objects – stars, planets, galaxies – are formed by accretion.

Accretion disk is a structure fomed by matter that is in orbital motion around the massive body

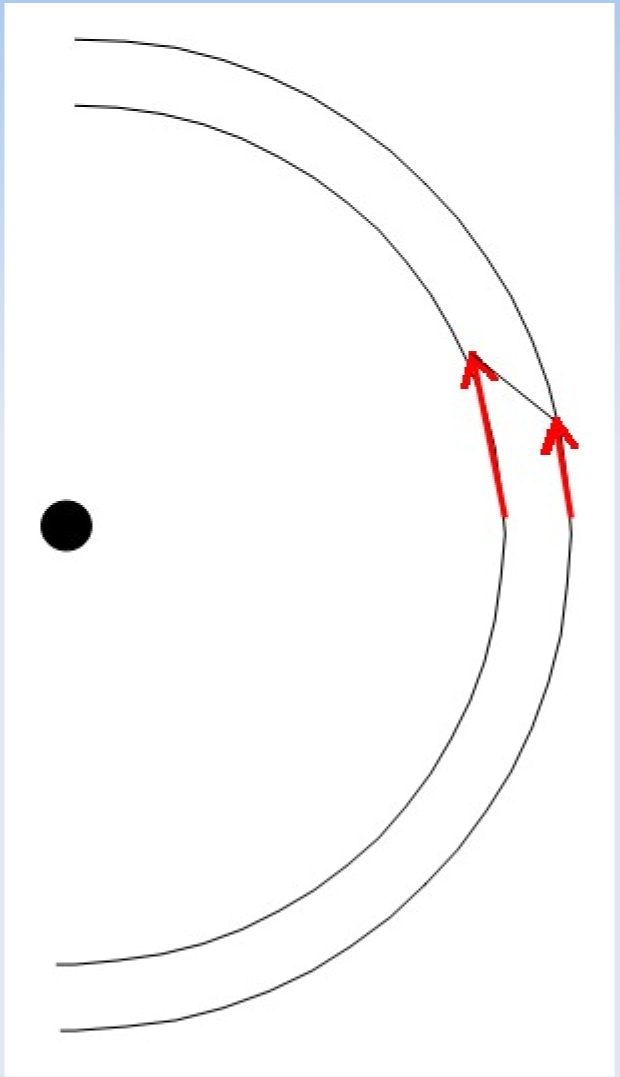


Formation of an accretion disk

- Ring of width dR rotates differentially
- This leads to a shear stress

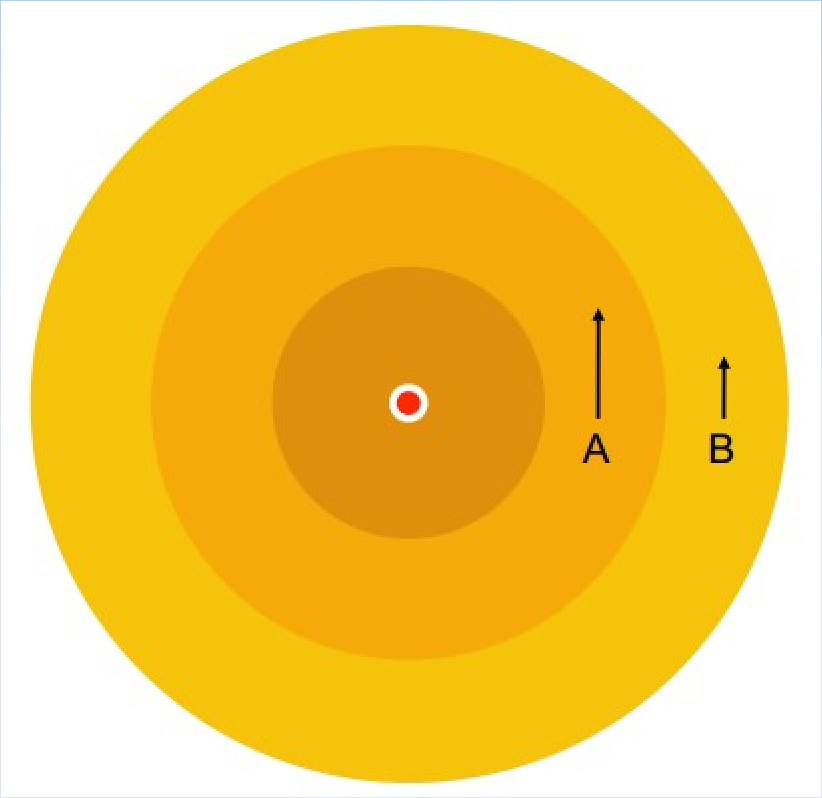
$$\Delta V = \frac{V}{2}\frac{\Delta R}{R}$$

- Friction opposes shear and causes the ring to spread inward and outward
- Angular momentum is transported outwards



Formation of accretion disk

- Subsequent rings are loosing angular momentum, but they are forced to stay on Keplerian orbits
- Rings are moving inwards
- Gravitational potential energy of the gas is liberated and disk heats up
- Viscosity: responsible for angular momentum transport and disk heating



Viscous disk

- Viscous torque is exerted by the outer ring on the inner (and vice versa)

$$G(R) = 2\pi R \nu \Sigma R^2 \frac{d\Omega}{dR}$$

where $\nu = \lambda v$ is the kinematic viscosity, with λ given by the mean free path and v is thermal speed, for molecular transport, or the wavelength and speed of turbulence.

- Dissipation rate is given by the net torque per unit area,

$$D(R) = \frac{G}{4\pi R} \frac{d\Omega}{dR} = \frac{9}{8} \nu \Sigma \frac{GM}{R^3}$$

where we integrated over disk thickness, and $\Sigma = \rho H$ is the surface density.

Structure of the disk

- Disk is optically thick, geometrically thin.
- Height averaging → integrated (surface) density
- Stationary: time derivatives vanish
- We solve the equations of radial mass conservation and momentum conservation (Euler equation).
- Adopt the proper inner boundary condition (on the star surface)

Temperature as a function of radius

- The mass accretion rate

$$\dot{M} = 2\pi R \Sigma v_r$$

- From the angular momentum equation for steady disk we have

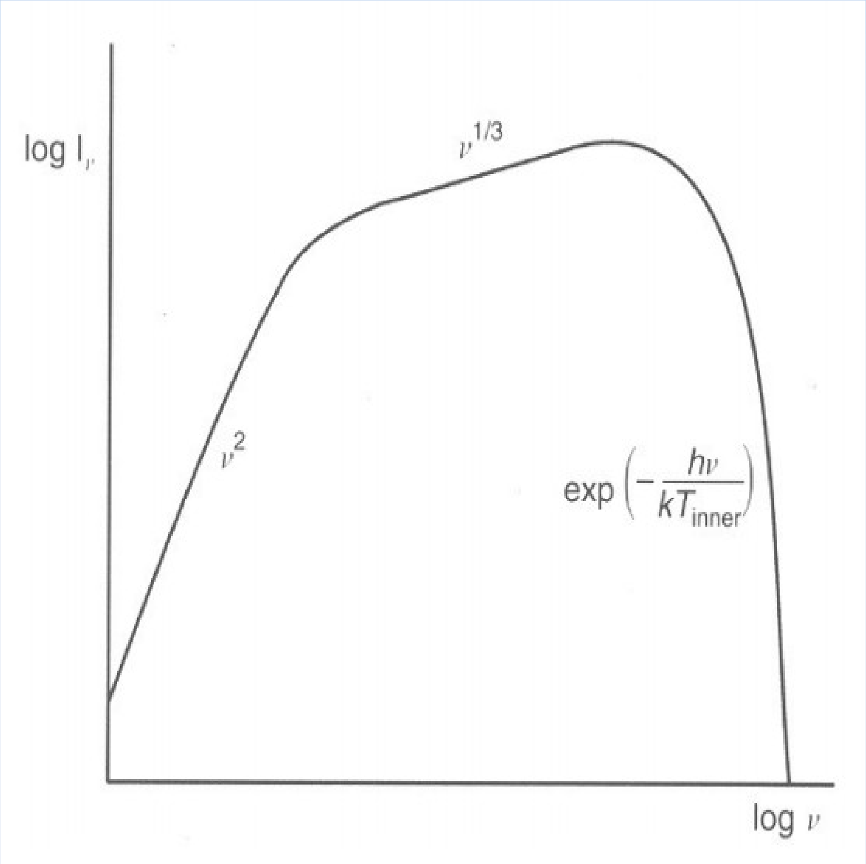
$$\nu \Sigma = \frac{\dot{M}}{3\pi}\left[1 - \sqrt{\frac{R_{star}}{R}}\right]$$

- Therefore the dissipation rate, so the locally emitted energy flux, is given by

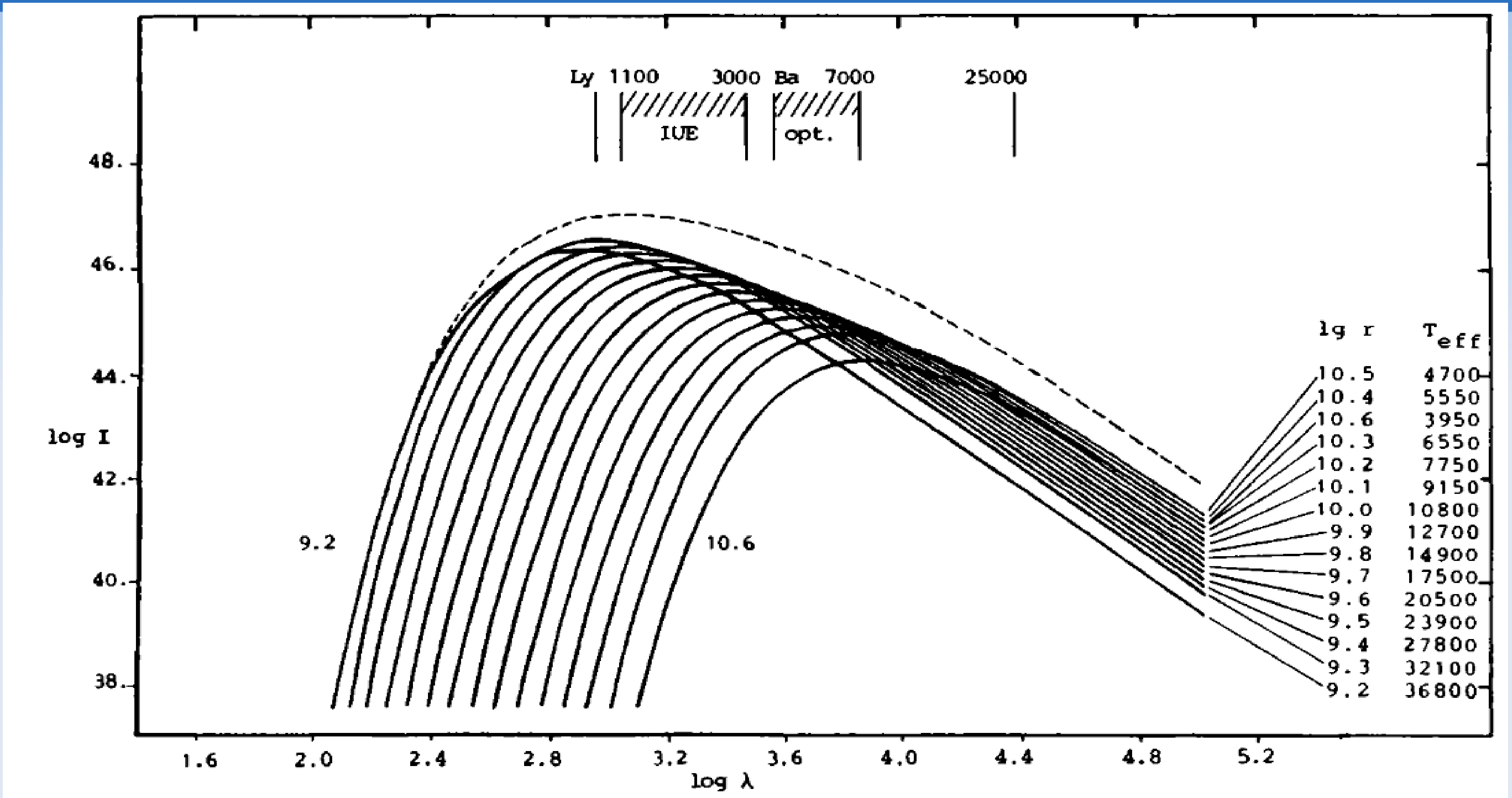
$$F_{tot} = \frac{3 G M \dot{M}}{8\pi R^3} f(R) = \sigma T(R)^4$$

Shape of the spectrum

- Temperature of the disk depends on radius.
- From the outer parts, we'll see the Rayleigh-Jeans tail of the spectrum
- From the inner edge, we'll see the exponential cut-off



Disk black body



Contributions from different annuli of the disk to its thermal spectrum

Temperature and spectrum

- Average effective temperature: blackbody approximation

$$L_{disk} = \pi R^2 \sigma T_{eff}^4$$

- Thus the disk temperature scales with fourth root of mass and accretion rate, and inveresly with root of disk size
- The temperature decreases with radius
- Larger disks are cooler

Disk energetics

- If the heat generated by viscosity is radiated, the luminosity of the disk is governed by the mass transfer rate and compactness of the accretor

$$L \propto \frac{G M \dot{M}}{R}$$

- Efficiency of this process is the fraction of the rest mass energy that is radiated

$$L = \eta \dot{M} c^2$$

Examples

- Main sequence star: $\eta = 2\text{x}10^{-6}$, (cf. The efficiency of H-He conversion is 0.007)
- White dwarf $\eta = 10^{-4}$
- Neutron star $\eta = 0.2$
- Schwarzschild black hole $\eta = 0.057$
- Extreme Kerr black hole $\eta = 0.43$

Break

Steady disk

- External conditions may change on timescales much longer than viscous. Therefore we can neglect time derivatives and

$$R\Sigma V_R = const = -\frac{1}{2\pi}\dot{M}$$

- From the angular momentum equation, we get

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1-\left(\frac{R_{star}}{R}\right)^{1/2}\right]$$

where the term in brackets comes from the constant, determined by inner boundary condition (it is not valid if the star has a strong magnetic field, or rotates much faster than $\Omega_K(R_*)$).

Structure in the vertical direction

- Hydrostatic equilibrium (given by the z-component of the Euler equation, with neglected velocity terms)

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = -\frac{GM}{r^2}\frac{z}{r}$$

- For a thin disk, z<<R, H<<R, z~H, dP/dz ~ P/H
- The local Kepler velocity is highly supersonic:

$$H \approx c_s/\Omega^{2,} \text{ so } c_s << (GM/R)^{1/2}$$

- The radial drift velocity is subsonic

$$v_R \sim \alpha\, c_s\, H/R << c_s$$

Local structure of thin disk

- Vertical structure is decoupled from radial and is treated as 1-d version of stellar structure
- Vertical energy transport may be radiative or convective, depending on temperature gradient
- For radiative transport, the flux through z=const surface (in plane-parallel approximation) is

$$F(z) = -\frac{16\sigma T^3}{3\kappa\rho}\frac{\partial T}{\partial z}$$

with the Rosseland-mean opacity, given by Kramer's

$\kappa(\rho,T) = 5\mathrm{x}10^{24}\ r\ T^{-7/2}\ [\mathrm{cm}^2\ \mathrm{g}^{-1}]$

- The disk is optically thick if $\tau = \kappa\rho H = \kappa\Sigma >> 1$.

Energy balance

- The energy balance requires:

F(z=H) - F(z=0)= D(R)

- If surface temperature is much smaller than central temperature, this will be

$$\frac{4\sigma T_c^4}{3\tau} = \frac{3 G M \dot{M}}{8\pi R^3}\left[1-\left(\frac{R_{star}}{R}\right)^{1/2}\right]$$

- The closing relation is the equation of state, and in general the pressure is a sum of gas and radiation pressures:

$$P = \frac{k}{\mu m_p}\rho T_c + \frac{4\sigma}{3c} T_c^4$$

with mean molecular weight μ~1 for neutral hydrogen

Magnitude of viscosity

- Viscous stresses are generated via thermal and turbulent motions
- In cylindrical coordinates, the viscous stress tensor $r\phi$ component is

$$T_{r\phi} = \rho\, \nu\, r\, \Omega'$$

- The kinematic viscosity, according to Shakura & Sunyaev (1973) prescription, is

$$\nu = \alpha\, c_s\, H$$

with a constant $\alpha \sim 0.1$.

Steady state disk solutions

- $\Sigma = 5.2\ \alpha^{-4/5}\ dM_{16}^{\ 7/10}\ m_1^{\ 1/4}\ R_{10}^{-3/4}\ f^{\ 14/5}$ [g cm^{-2}]
- $H = 1.7\mathrm{x}10^8\ \alpha^{-1/10}\ dM_{16}^{\ 3/20}\ m_1^{\ -3/8}\ R_{10}^{\ 9/8}\ f^{\ 3/5}$ [cm]
- $\rho = 3.1\mathrm{x}10^{-8}\ \alpha^{-7/10}\ dM_{16}^{\ 11/20}\ m_1^{\ 5/8}\ R_{10}^{\ -15/8}\ f^{11/5}$ [g cm^{-3}]
- $T_c = 1.4\mathrm{x}10^4\ \alpha^{-1/5}\ dM_{16}^{\ 3/10}\ m_1^{\ 1/4}\ R_{10}^{\ -3/4}\ f^{\ 6/5}$ [K]
- $\tau = 190\ \alpha^{-4/5}\ dM_{16}^{\ 1/5}\ f^{\ 4/5}$
- $\nu = 1.8\mathrm{x}10^{14}\ \alpha^{\ 4/5}\ dM_{16}^{\ 3/10}\ m_1^{\ -1/4}\ R_{10}^{\ 3/4}\ f^{\ 6/5}$ [cm^2 s^{-1}]
- $V_R = 2.7\mathrm{x}10^4\ \alpha^{\ 4/5}\ dM_{16}^{\ 3/10}\ m_1^{\ -1/4}\ R_{10}^{\ -1/4}\ f^{\ -14/5}$ [cm s^{-1}]

Shakura & Sunyaev (1973). Assumed μ=0.615 and no radiation pressure. $R_{10}=R/(10^{10}$ cm), $m_1=M/M_{Sun}$, $dM_{16}=dM/(10^{16}$ g s^{-1}); f is given by the boundary condition at inner edge.

Outer edge of the disk

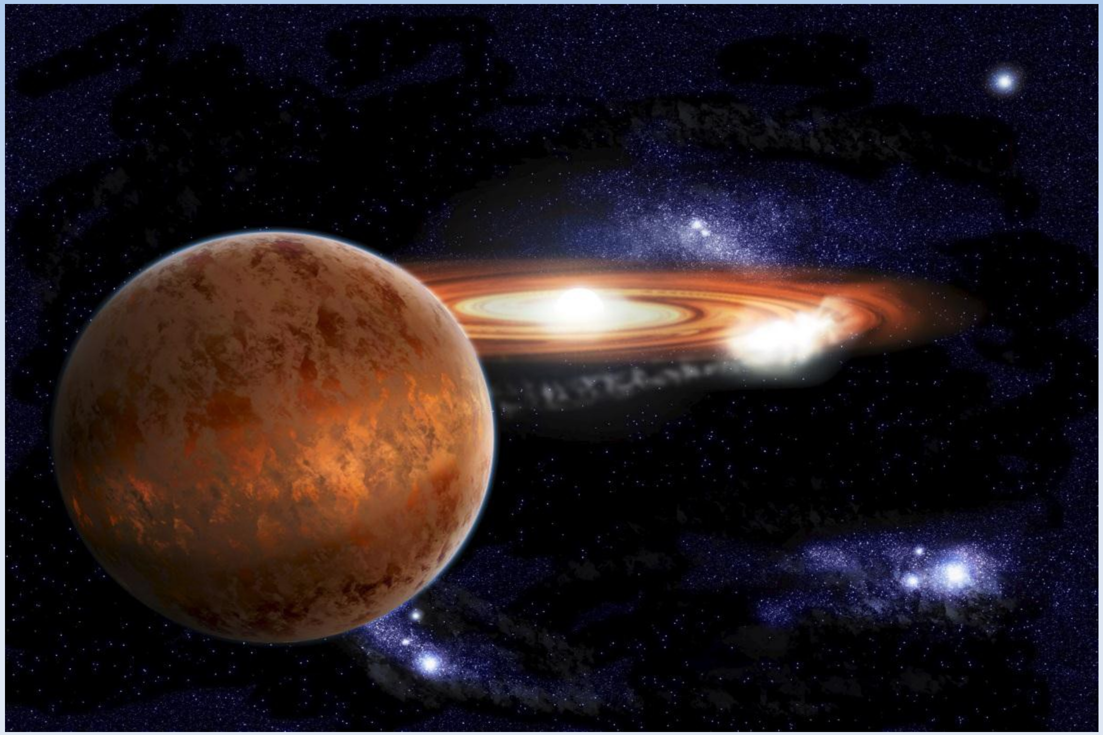
- Tidal interaction with the companion star keeps the disk from overflowing the Roche lobe.
- Paczyński (1977); Papaloizou & Pringle (1977)

$$\frac{R_{max}}{a} = \frac{0.60}{1+q}$$

- Combined effects of viscous diffusion, tidal dissipation and mass transfer stream: outer edge could reach up to 80-90% of the Roche lobe radius

Hot spot

- The stream of gas heats the outskirts of accretion disk with supersonic speed
- The shock-heated spot may radiate in Optical band and emit more than the donor star and disk itself

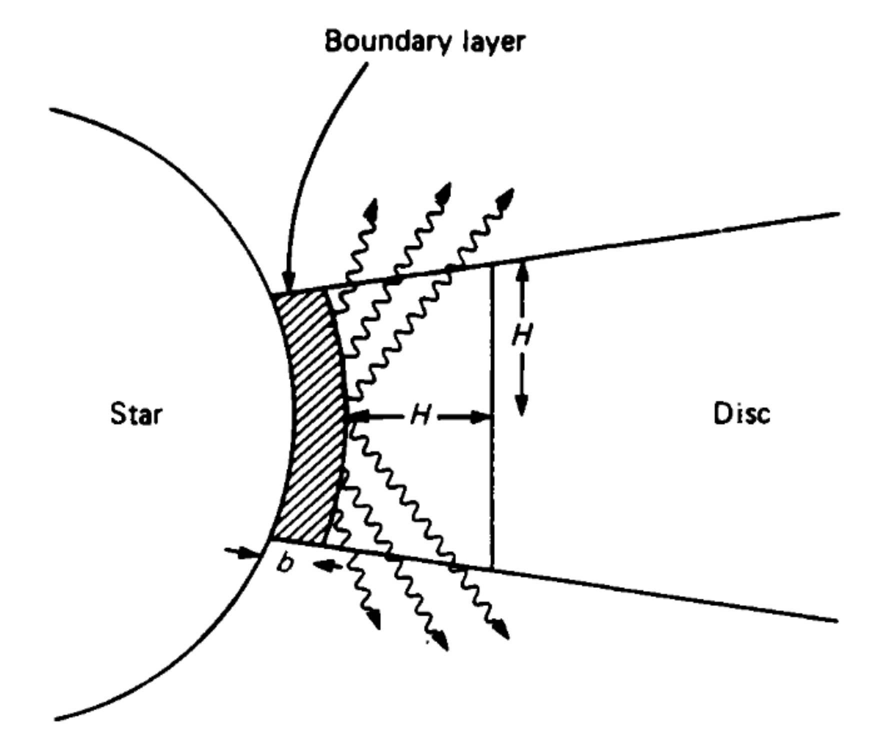


Inner edge of the disk

- Star's surface
- Black hole: radius of the marginally stable circular orbit, depends on the spin parameter
- Detailed discussion in Krolik & Hawley (2002) describes other 'working' definitions:
  - Radiation edge: innermost radius from which ignificant luminosity emerges; different from r_{ms} due to e.g. Gravitational redshift, photon trapping
  - Reflection edge: material even inside rms can reflect and reprocess X-rays
  - Stress edge: magnetic stress may continue well inside r_{ms}
  - Turbulence edge: MHD turbulence ceases

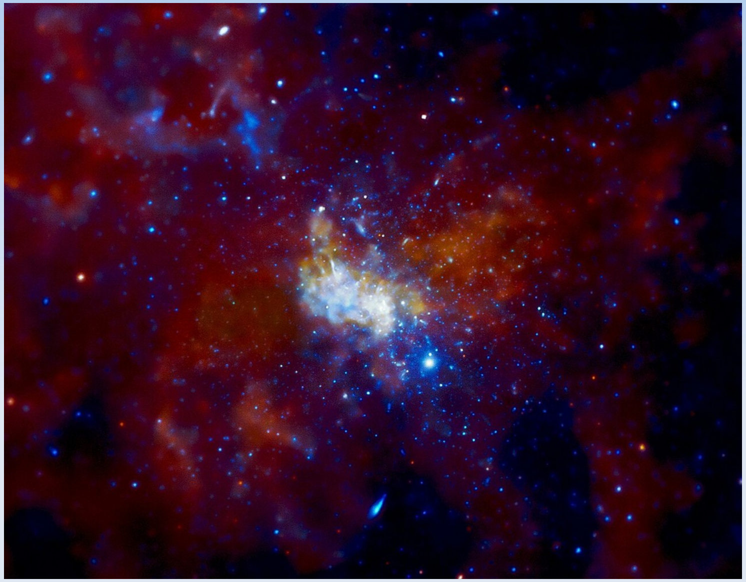
Boundary Layer

- Hard surface of the star (neutron star, white dwarf)
- Gas moving with Keplerian velocities in the disk must be decelerated to match the star's rotation
- Energy is used to spin up the star but is also dissipated
- Bulk of the boundary layer radiation is emitted in UV and X-rays



Spherical accretion

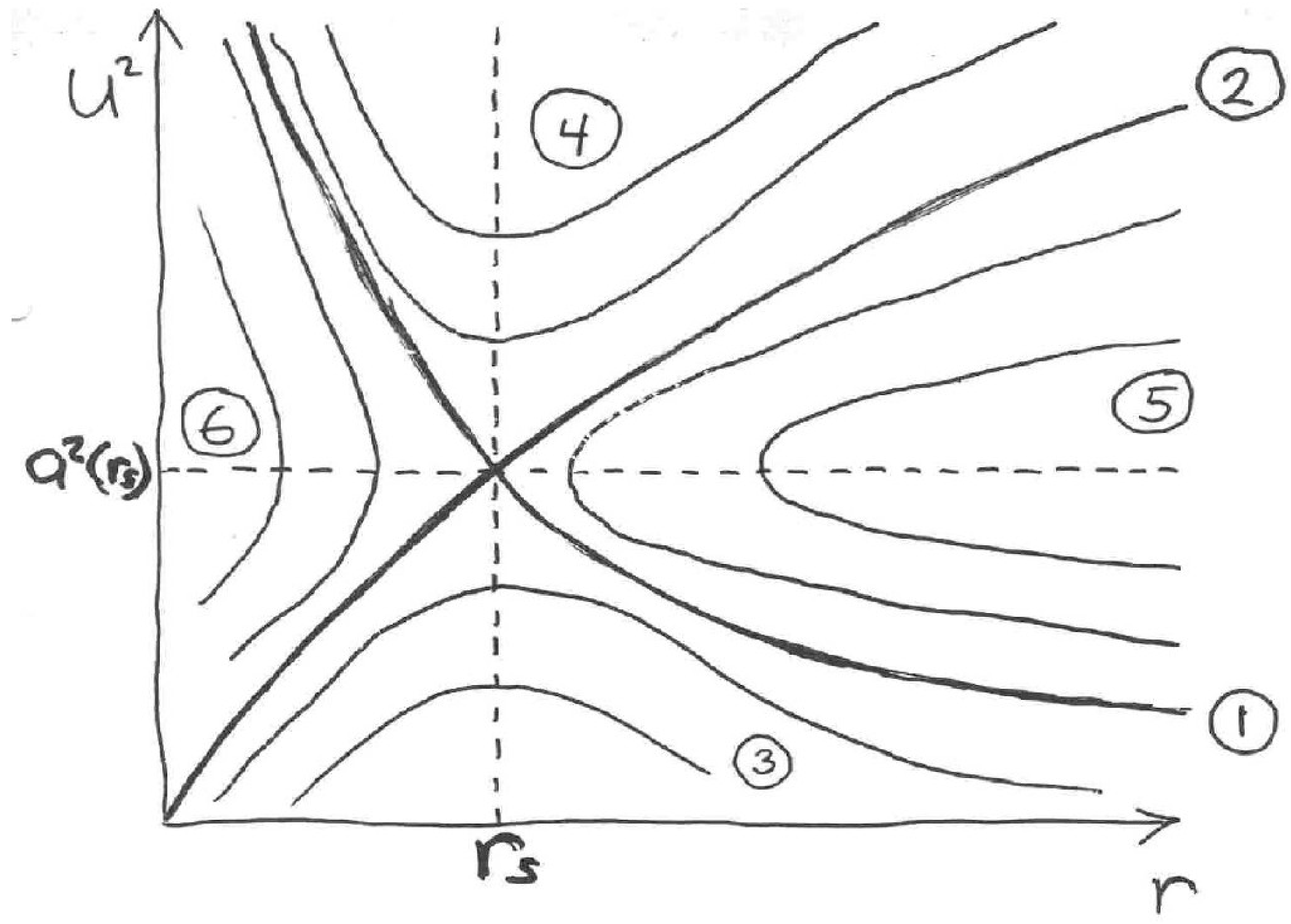
- Matter falls spherically onto central object (zero, or very low angular momentum)
- Infalling gas consists of ionised hydrogen
- Protons and electrons interact via Coulomb collissions
- If the mean free path is small, hydrodynamic approach is valid
- For collissionless plasma, hydrodynamics breaks up, for low accretion rates
- Magnetic fields may help change mean free path



Spherical accretion

- In hydrodynamic approach, the properties of gas are governed by the equations of fluid mechanics.
- Continuity equation enforces the conservation of mass
- Euler equation considers forces due to pressure gradient and gravity
- Bondi (1952) solved the problem in spherical symmetry, assuming polytropic equation of state

Topology of solutions



Bondi accretion

- Bernoulli equation

$$\frac{1}{2}v_r^2 + \frac{c_s^2}{\gamma - 1} - \frac{GM}{r} = const$$

- Problem can be parameterized by the sound speed and gas density at infinity

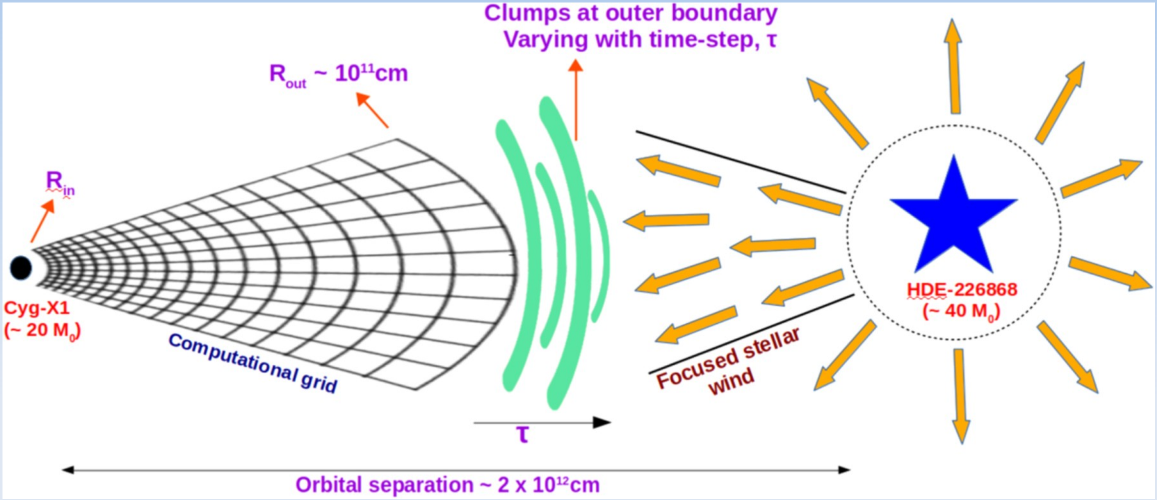
$$c_s^2(r_s) = c_{s,\infty}^2 \frac{2}{5 - 3\gamma}$$

$$\dot{M} = 4\pi\rho_\infty \left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}} \frac{G^2 M^2}{c_{s,\infty}^3}$$

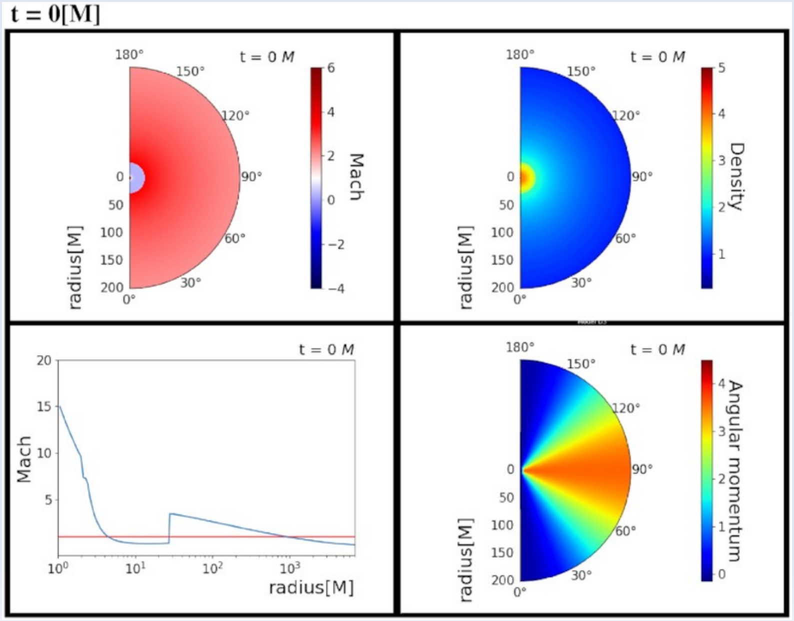
where $r_s = GM/(2\, c_s^2)$ is sonic radius

Examples

- Weakly active galaxy
- Wind accretion in binary system
- Adding a small angular momentum complicates the solution topology



Palit, I., et al. (2019; 2020)



Next week

- Observations of accretion disks in X-rays
- Radiative processes, soft and hard spectra

- Suggested literature:
- Frank J. et al. ”Accretion Power in Astrophysics”,
- Shapiro, Teukolsky – appendix G, spherical accretion onto black hole