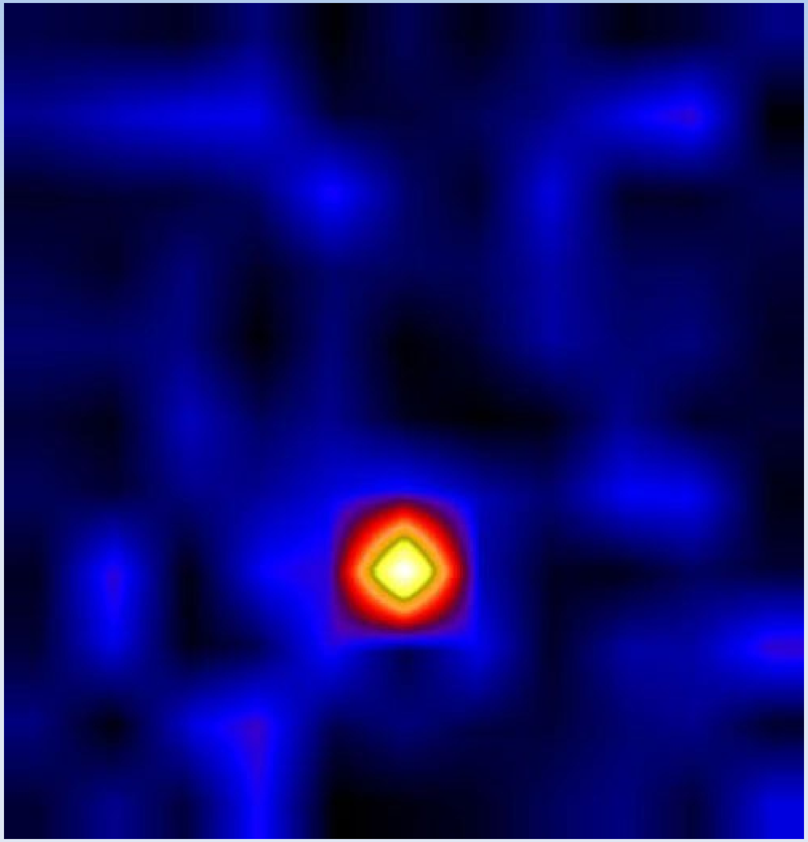
Compact Stars



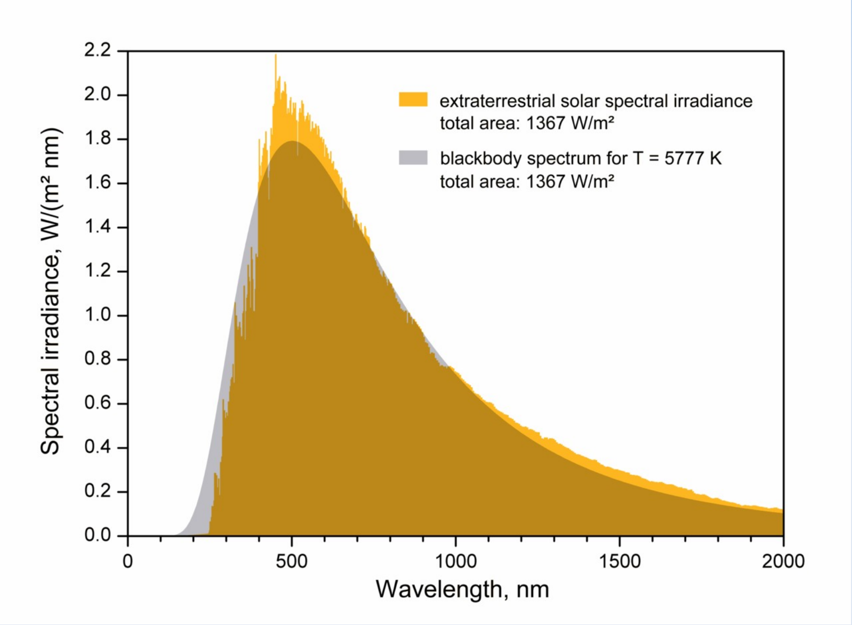
Lecture 2

Summary of the previous lecture

- We have talked about binary stars in general:
  - Visual, astrometric, spectroscopic, and eclipsing binaries
  - Motion around the center of mass, Kepler's laws, radial velocity curves
  - Determination of masses and radii of the components. Mass function as a lower limit for mass of the unseen companion
- We talked also about stellar classification, and mass-luminosity relation for a single star.
  - Main Sequence, spectral types.
  - Evolution of stars after Main Sequence, types of compact remnants

Effective temperature

- Effective temperature of a star is the temperature of black body which from 1 cm^2 of its area radiates out the same total energy at all wavelengths as the observed star.
- The surface brightness of a star is its black body intensity, wavelengths integrated, and emitted from a unit area of its surface
- The Stefan-Boltzmann law relates the surface brightness with temperature, σT^4



Stellar structure model

- Gravity force balanced the presure gradient

$$G\frac{M(r)}{r^2} = -\frac{1}{\rho(r)}\frac{dP}{dr}$$

where M*(r)* is mass enclosed within radius *r*. Pressure must decrease outwards.

- Thermal pressure in gas

$$P = n k_B T = \frac{\rho}{m_{molec}} k_B T$$

where *n* is number density of particles per unit volume, and m_{molec} is the mean molecular weight

Temperature profile

- Temperature of the center may be estimated from Virial Theorem.
- We have U = -2K

  where $U=GM^2/R$ (potential energy), $K=3/2Nk_BT$ (kinetic energy) and $N=M/m_{H.}$
- For the mass and radius of the Sun, this gives $T_c=10^7$ K. It is much higher than effective temperature of the surface.
- Energy generated in the center is transported out by neutrinos, produced in nuclear reactions, and by photons.
- Photons are scattered, and their mean free path is only 1 mm. It takes about 50000 yrs for a photon to escape from the Sun!

Radiation transport

- Every volume element in the star radiates as a black body.
- Higher energy density at smaller radii implies a net flow of radiation outward.
- It is described by diffusion equation, where l=$(\kappa\rho)^{-1}$ is the mean free path

$$\frac{L(r)}{4\pi r^2} = -\frac{c\,l}{3} 4aT^3 \frac{dT}{dr}$$

- Hence,

$$\frac{dT(r)}{dr} = -\frac{3L(r)\rho(r)\kappa(r)}{16\pi r^2 a c T^3(r)}$$

Conservation of mass and energy

- To complete the stellar structure model, we need to solve also the mass conservation equation

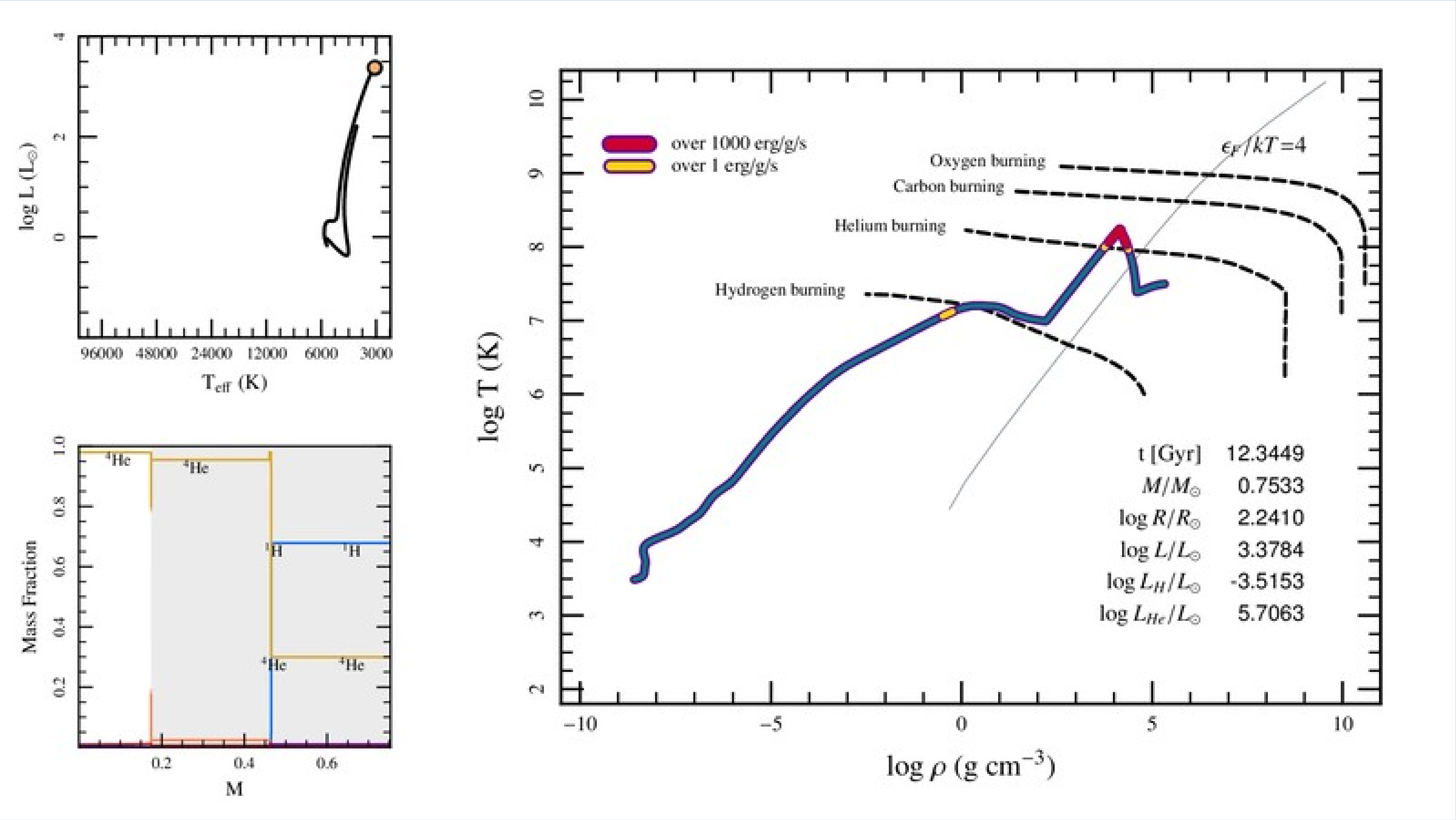
$$\boxed{\frac{d\,M(r)}{dr} = 4\pi r^2 \rho(r)}$$

- and energy conservation

$$\boxed{\frac{d\,L(r)}{dr} = 4\pi r^2 \rho(r)\,\epsilon(r)}$$

where ε is the power generated per unit mass. It depends on the composition

Solar lifetime with MESA code

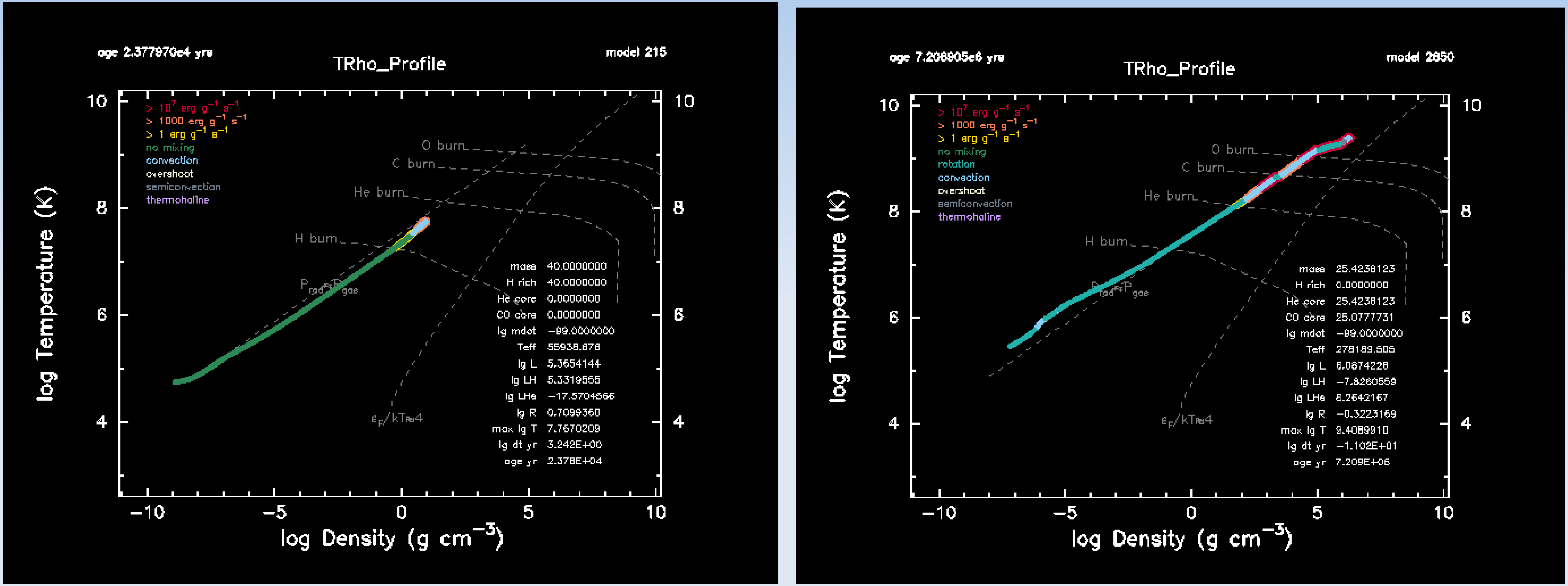


Courtesy: dr Josh Schwab, https://yoshiyahu.org/

Massive star evolution

- Medium mass stars evolving to WD, become more red on the RGB, and evolve horizontally on HR diagram.
- Star will exhaust He in the core, contract and eject PN.
- Massive stars: luminosity scales with $M^{3\text{-}4}$, so nuclear timescale scales as t~M/L~$M^{-2.5.}$
- After Si burning, further fusion unable to generate energy.
- The star will end up as core-collapse SN.
- The end product is sensitive to mass loss rate. This process is unstable and may be through eruptions or clumpy winds (Smith et al. 2014).
- Most massive stars (>100 M_{Sun}) are unstable due to radiation pressure and formation of electron-positron pairs.

Evolution of a massive star

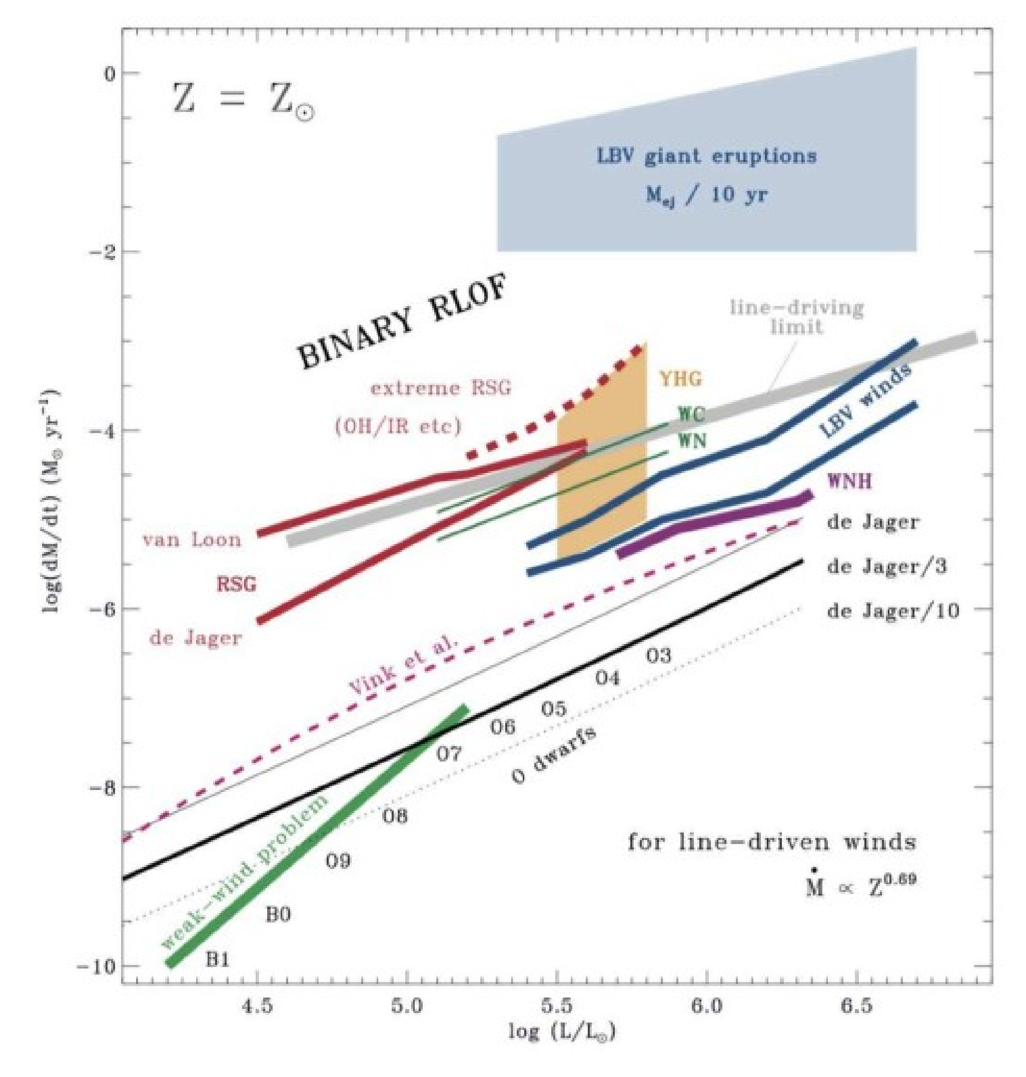


Profiles of density and temperature in the star on the main sequence (left) and evolved (right). The mass of the star on ZAMS was 40 M_{Sun}. Evolved star (age $7x10^8$ yrs) mass is reduced to 25 M_{Sun}, due to mass loss. Star is burning carbon and Oxygen in the core.

Code: *https://docs.mesastar.org*

Mass loss

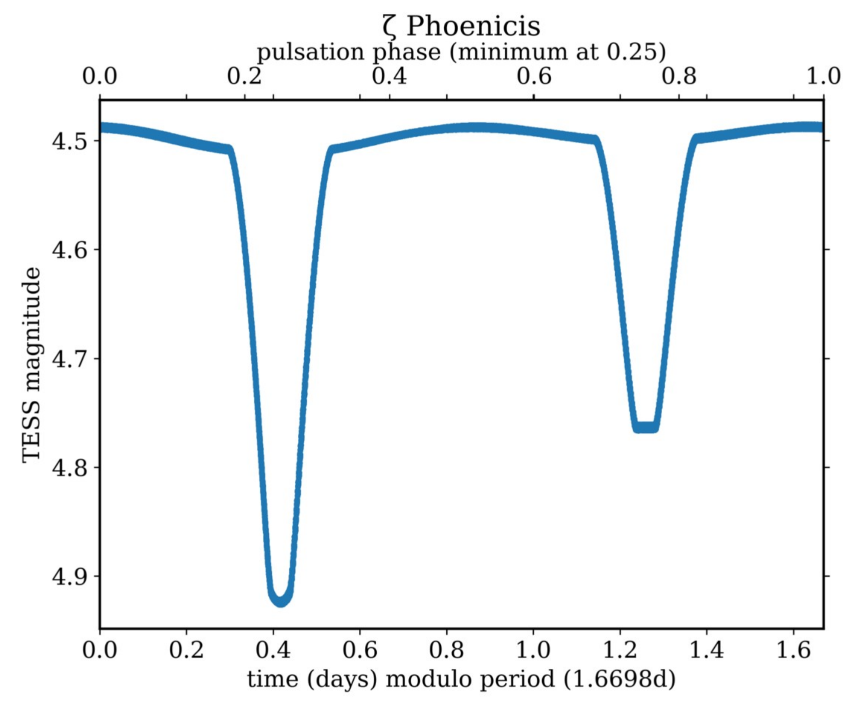
- In the early universe, at lower metallicity the mass-loss was less efficient.
- At very low metallicities the mass-loss is determined by very weakly bound sites in the outer atmosphere, where parcels of gas are carried away by momentum exchange between the outgoing flux and gas due to opacity.
- If the gas is only H, then the opacity is low because there are very few atomic transitions. So we expect more massive stars early on.
- These most massive stars may not go through the normal channels to end up at core-collapse SN.
- Instead they may have a significant pair-instability and even complete disruption through a supernova leading to no remnant.



Algols

Algol system prototype: beta Persei (discovered in 1669)

The Algol paradox: more massive component Algol Aa1 is still in the main sequence, but the less massive Algol Aa2 is a subgiant star at a later evolutionary stage.



Now the class has over 3500 members

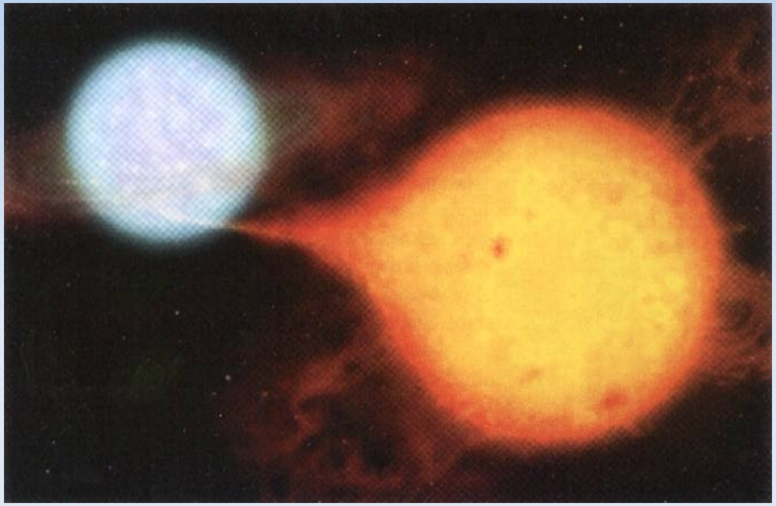
They are eclipsing binaries, some of them are MS+MS and other are MS+evolved star

Interacting binaries

- Interacting = close binaries. Between the components occurs significant interaction, other than gravitational attraction between the point masses.
- Radiative interaction: heating one component's surface by the hotter companion
- Tidal distortion: combination of gravitational and centrifugal effects

Algol system

- Algol is a semidetached binary where the primary component is an early type, main sequence star that does not fill its Roche lobe, while the cooler, fainter, larger, less massive secondary component lies above the main sequence in a Hertzsprung–Russell diagram and fills the Roche lobe.



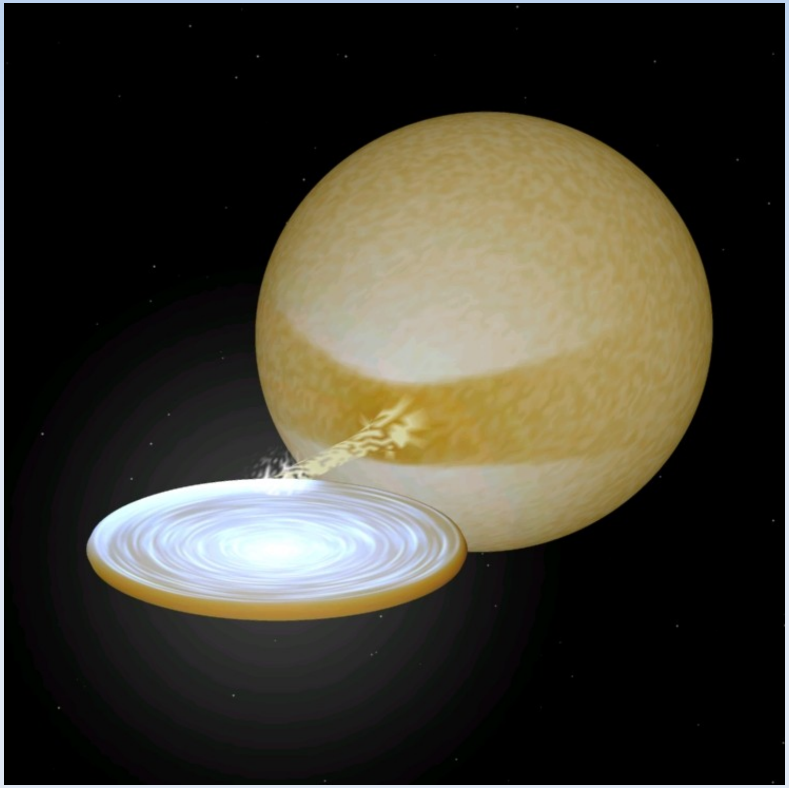
- Algol Aa2 is a orange-red star of spectral and luminosity type K0-IV. It has become a cool low-mass subgiant in which tidal forces from massive B8 type, massive companion Aa1 have distorted the swollen, outer gas envelope of the star into a teardrop shape

Lesson from the Algol Paradox

- The Algol paradox may be solved by the mass transfer between the binary components
- Some binaries might have gone thorugh the phase of common envelope (contact binaries)
- Formerly more massive star is now less massive, but still more evolved

Mass transfer and accretion disks

- Mass transfer problem was motivated by the observational paradoxes, e.g. the Algol paradox
- The phenomenon of mass transfer may occur in semi-detached systems
- The mass may be transported via accretion disk
- Accretion through wind, and common envelope transfer, may also happen and be more complex

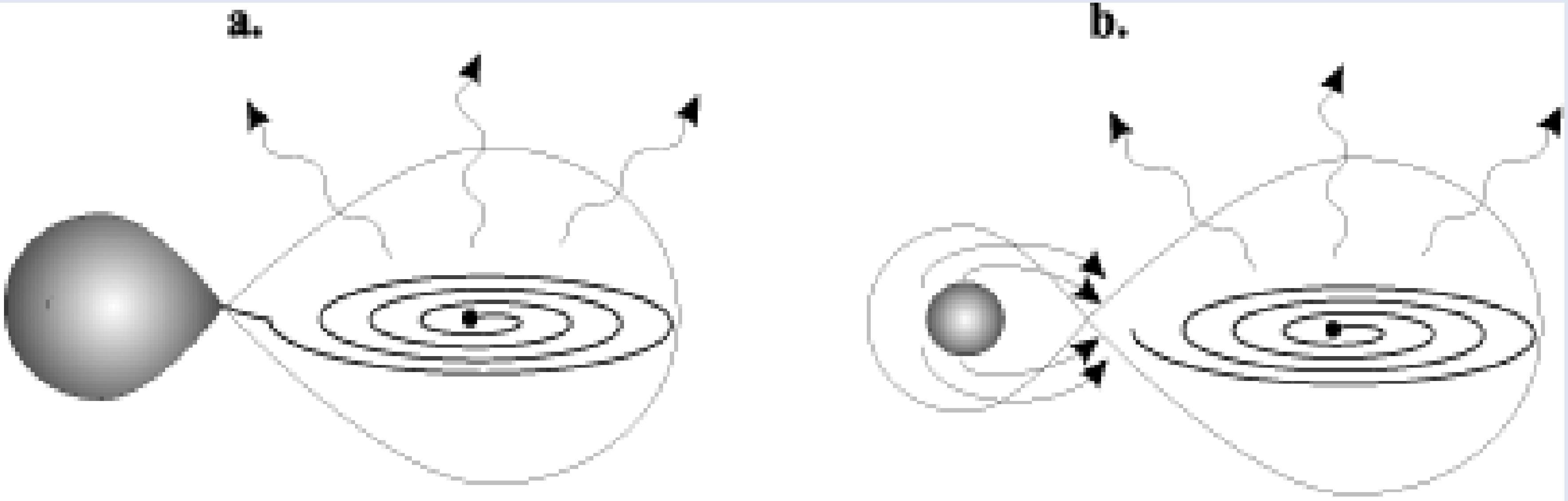


Example of mass transfer system in action

- Cataclysmic variables: small separation binaries, contain accreting white dwarfs.
- White dwarf is the end product of stellar evolution, but must be born iside the Red Giant star.
- Binary might have been wider, than separation shrinked, possible earlier common envelope phase.

Mechanisms of mass transfer in wide orbits

- Focused wind accretion (high mass X-ray binaries)
- Roche lobe overflow (low mass X-ray binaries)



Roche lobe overflow

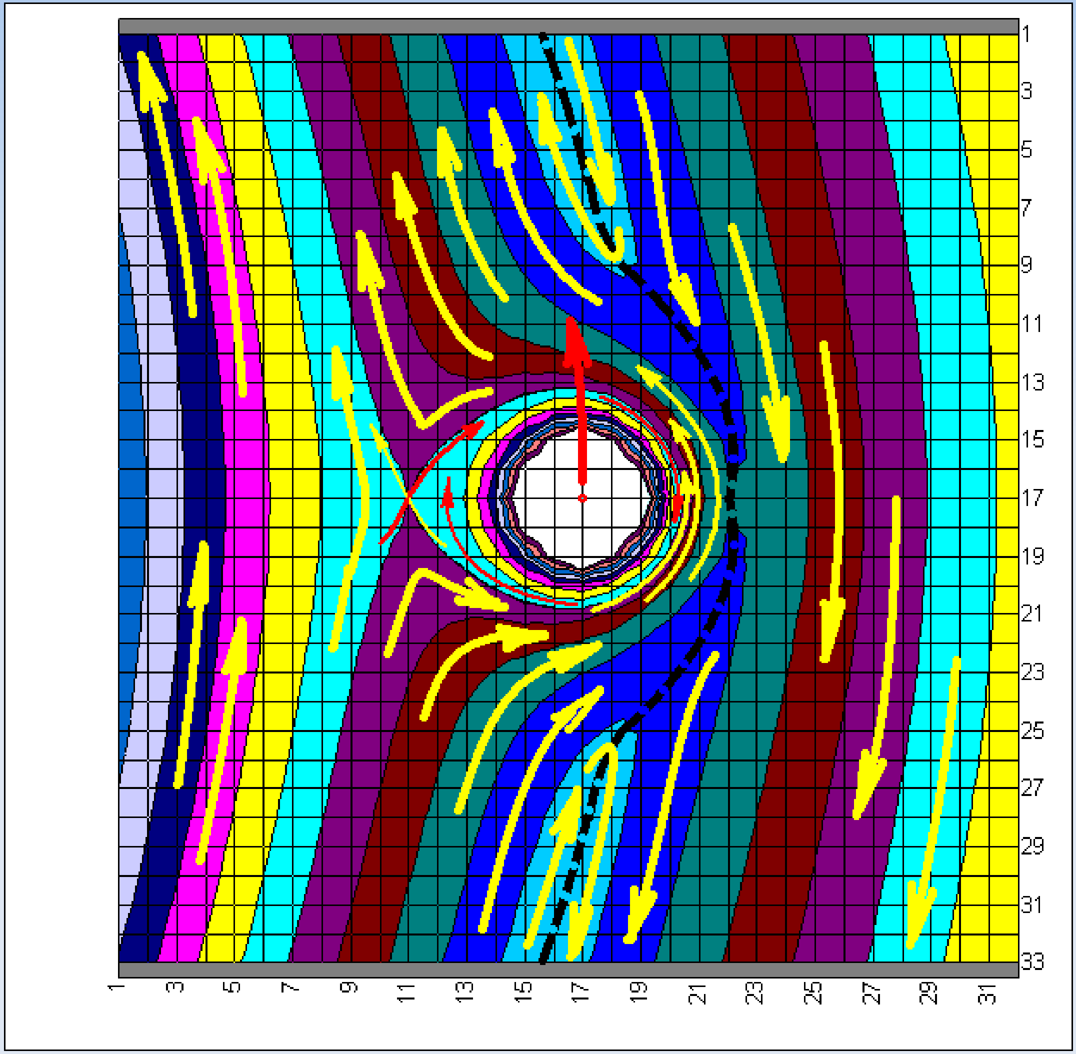
- Detached and semi-detached systems
- Star's surface is subject to rotational and tidal distorsion
- Shape of the surface is defined by the constant potential (sum of gravitational and centrifugal forces)

Break

Roche lobe. Definition

Roche lobe is the region around a star in a binary system within which orbiting material is gravitationally bound to that star.

Roche sphere approximates the gravitational sphere of influence of one astronomical body in the face of perturbations from another heavier body around which it orbits.



Roche model

Assumptions:

- Stars are on circular orbit and corotate with the orbital period
- Gravitational field is approximated by point masses

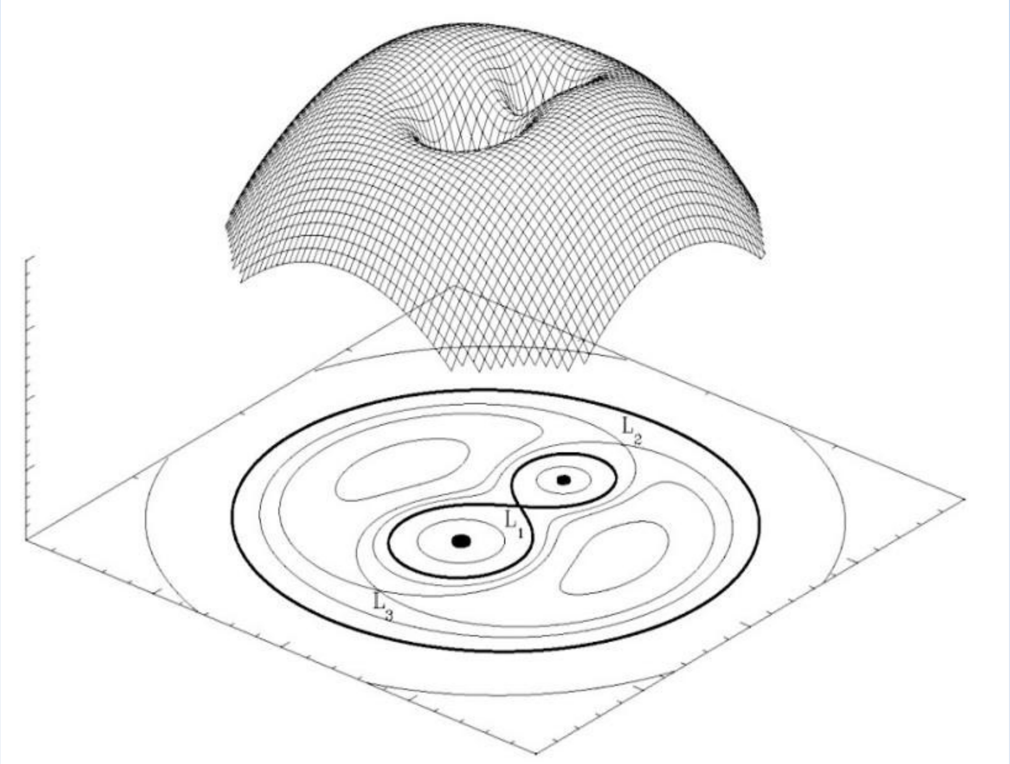
$$\Phi_{\mathrm{R}} = -\frac{GM(1)}{(x^2+y^2+z^2)^{1/2}} - \frac{GM(2)}{[(x-a)^2+y^2+z^2]^{1/2}} - \frac{1}{2}\Omega_{\mathrm{orb}}^2[(x-\mu a)^2+y^2]$$

Total potential in coordinate system with origin in the primary star;

$\mu = M_2/(M_1+M_2)$ is the reduced mass

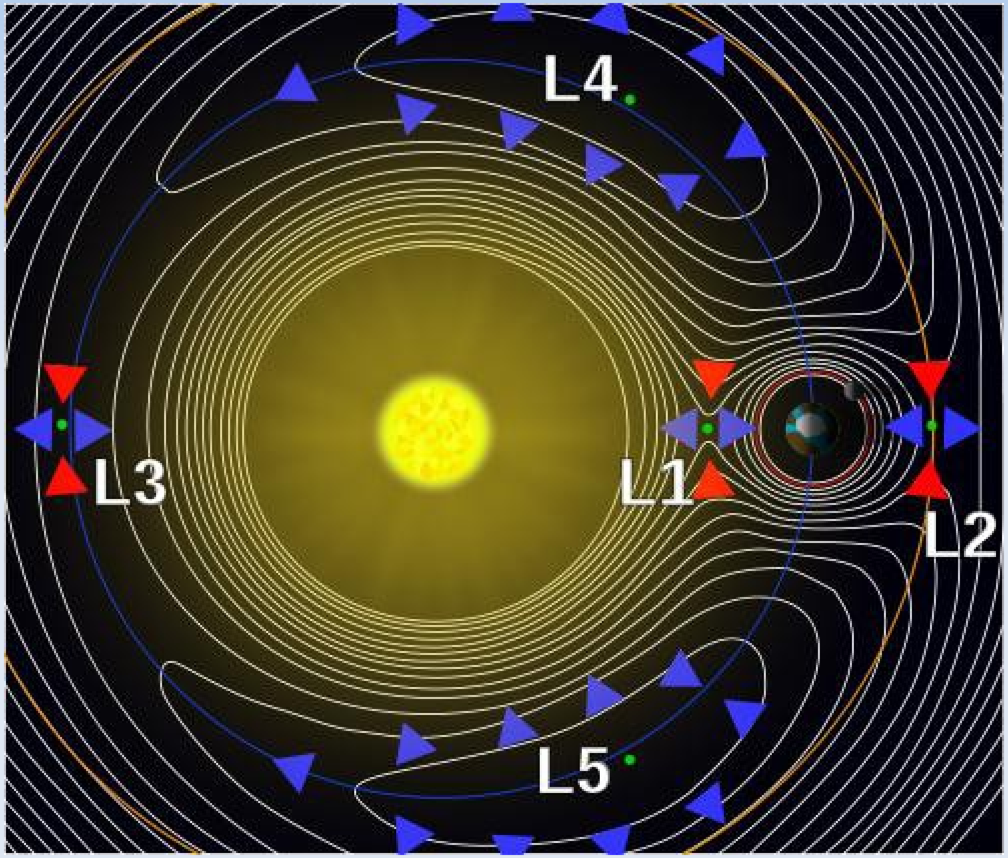
Roche model

- Shape of the equipotential surfaces depends on the mass ratio, q
- Their size scales with orbital separation, a
- Close to stars, they are nearly spherical
- Further, they are elongated due to tidal effect and flattened due to centrifugal force



Lagrange points

- L_1 – inner point, mass can flow from one star to the other
- L_2, L_3 – unstable points, matter can leave the system
- L_4, L_5 – stable points, in the top of the equilateral triangles formed by the centers of two stars



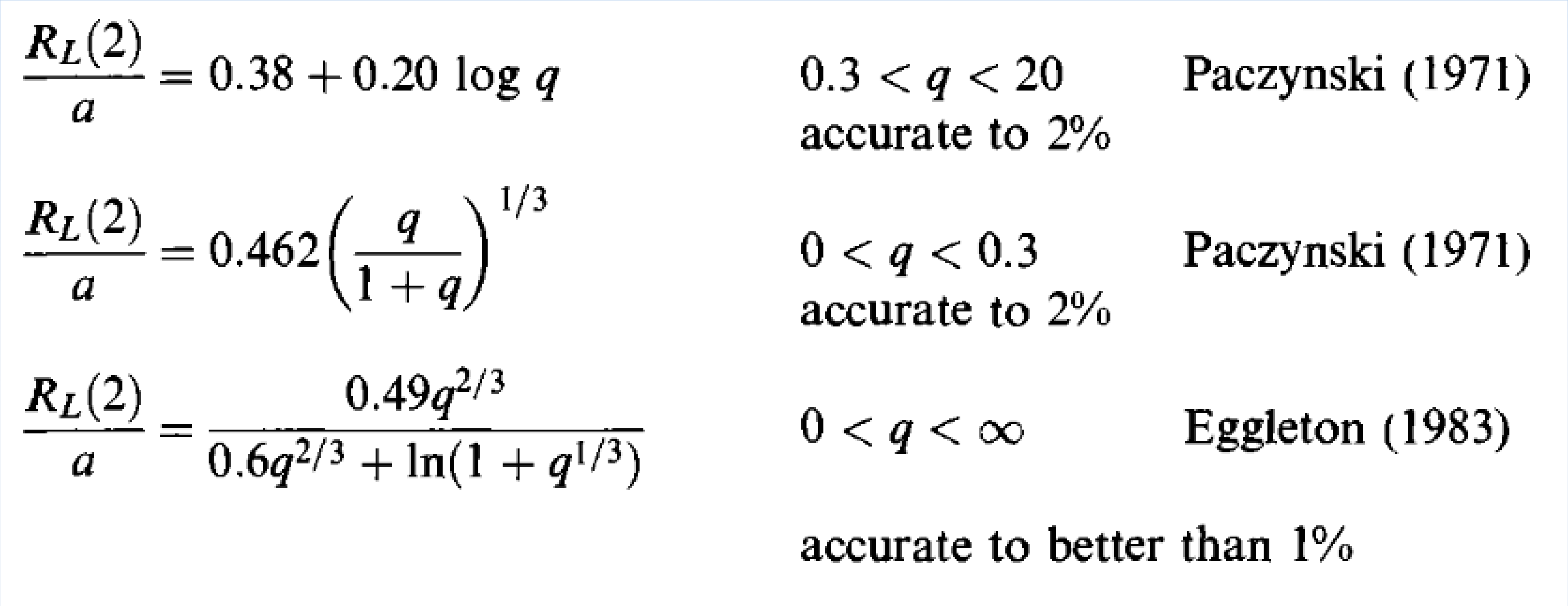
Roche lobe overflow

- Roche lobe of a star is the innermost equipotential surface that encloses both stars
- If the star fills its Roche lobe, the surface material is no longer gravitationally bound to that star
- In the inner Lagrangian point L_1, the material feels no net force: the gravity from the stars and centrifugal force cancel
- In consequence, the gas can escape from the star through the point $L_1$and is captured by the companion

Size of Roche lobe

- The accurate size of Roche lobe must be determined numerically
- Approximately, the radius of the closest sphere that encloses star's Roche lobe, may be estimated
- Accurate formulae have been derived by e.g. B. Paczyński (1971) – to 2%

Fitting formulae

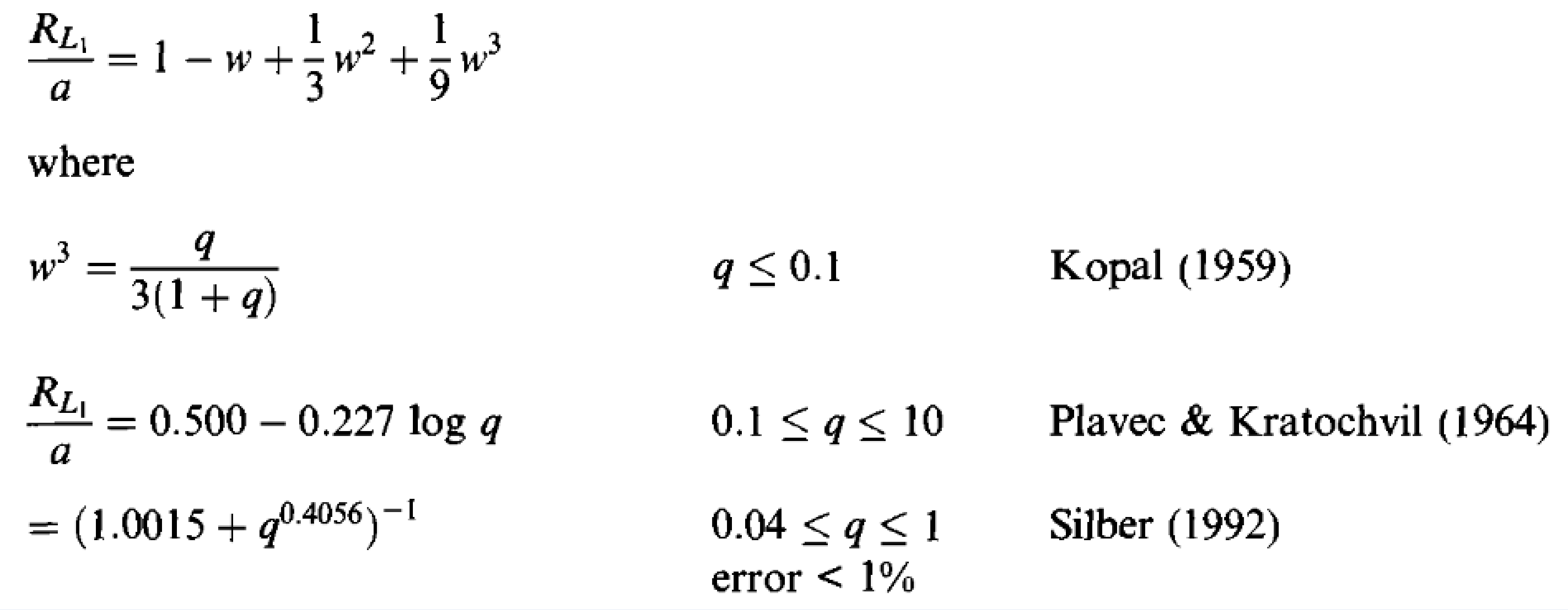


$\frac{R_L(2)}{a} = 0.38 + 0.20 \log q$	$0.3 < q < 20$ accurate to 2%	Paczynski (1971)
$\frac{R_L(2)}{a} = 0.462\left(\frac{q}{1+q}\right)^{1/3}$	$0 < q < 0.3$ accurate to 2%	Paczynski (1971)
$\frac{R_L(2)}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$	$0 < q < \infty$ accurate to better than 1%	Eggleton (1983)

The volume radius of the Roche lobe of the secondary star. Here a is orbital separation.

Mass ratio $q=M_2/M_1$

Distance to the point L1



$$\frac{R_{L_1}}{a} = 1 - w + \frac{1}{3}w^2 + \frac{1}{9}w^3$$

where

$w^3 = \frac{q}{3(1+q)}$	$q \le 0.1$	Kopal (1959)
$\frac{R_{L_1}}{a} = 0.500 - 0.227 \log q$	$0.1 \le q \le 10$	Plavec & Kratochvil (1964)
$= (1.0015 + q^{0.4056})^{-1}$	$0.04 \le q \le 1$ error < 1%	Silber (1992)

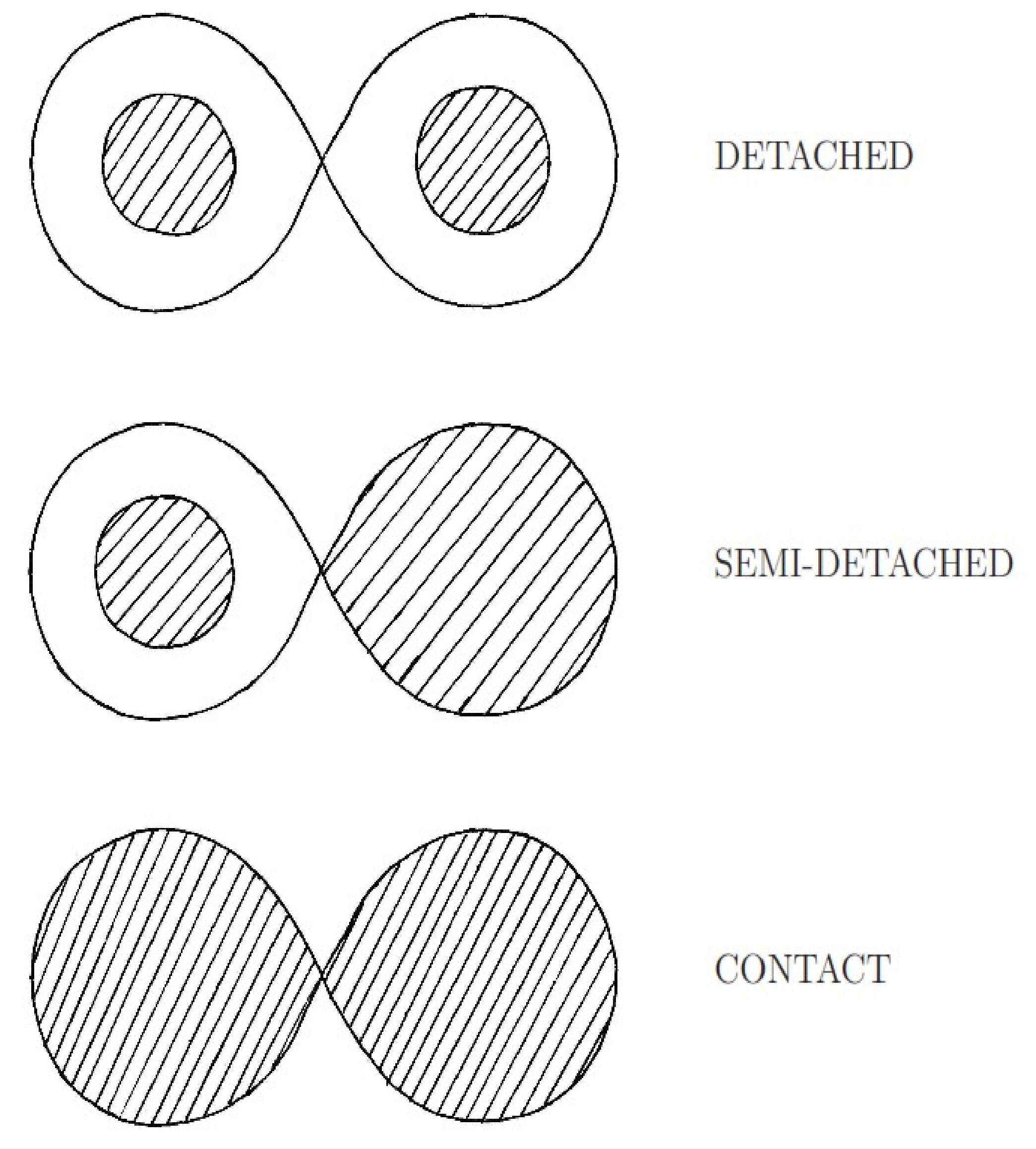
Measured from the center of the primary (mass gaining) star

Mass transfer

- Mass transfer starts when one of the stars fills its Roche lobe. This may occur when it expands (Red Giant phase), or when the binary system lost the angular momentum and the Roche lobe shrinked
- If mass ratio is small, the transfer is conservative. As the Roche lobe expands, the mass transfer is stopped, because the secondary star is no longer filling its lobe
- If mass ratio is large ($q>5/6$), the Roche lobe contracts and the transfer is more rapid

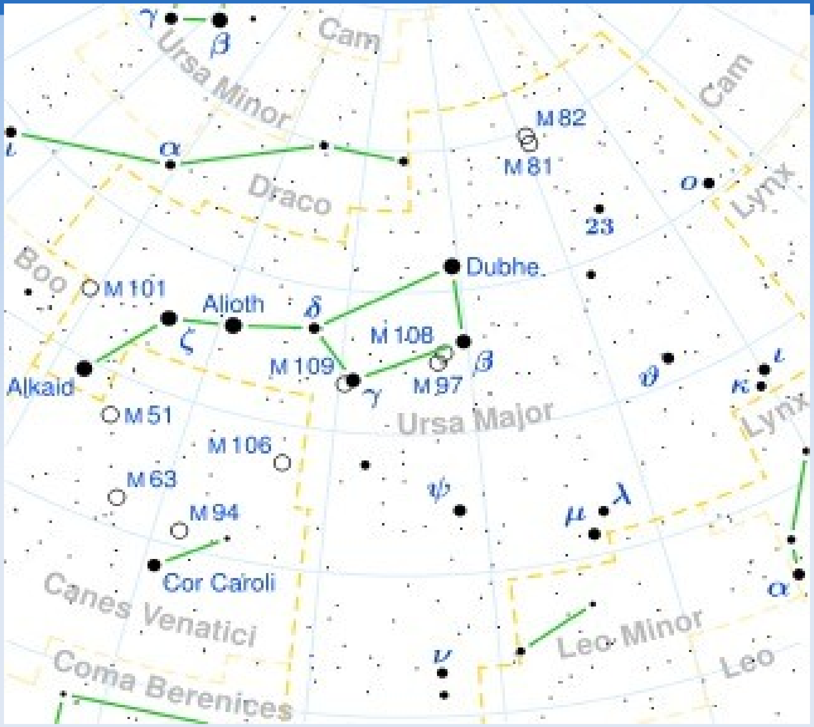
Binary systems classification

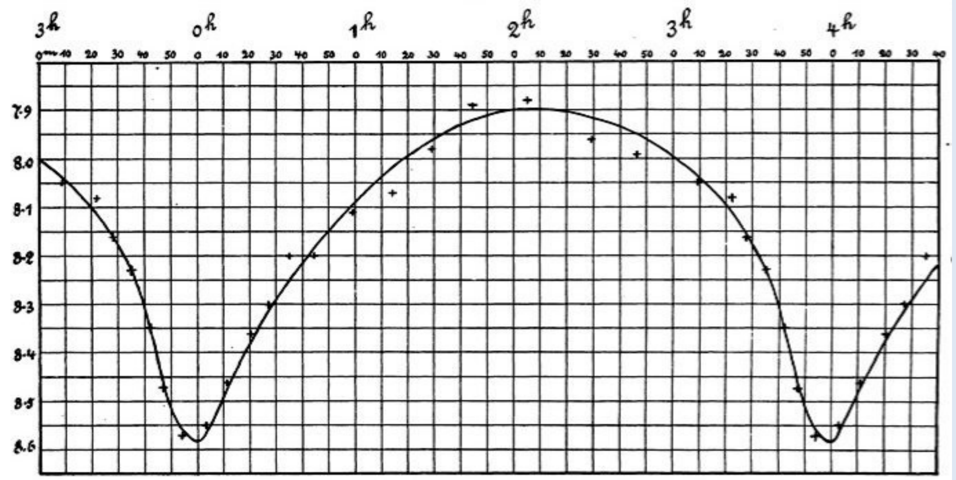
- Detached: stars are roughly spherical, neither fills its Roche lobe
- Semi-detached: one of the stars fills its Roche lobe and is not spherical
- Contact: both stars fill their Roche lobes



W Ursae Maioris

- Discovered in 1903
- Visual magnitude of 7.7
- Distance of 53 pc (from parallax measure)
- Variable, P=0.33 days
- Primary star of 1.19 M_{Sun} and 1.08 R_{Sun}, secondary: 0.57 M_{Sun} and 0.78 R_{sun}
- Thought to be a contact binary



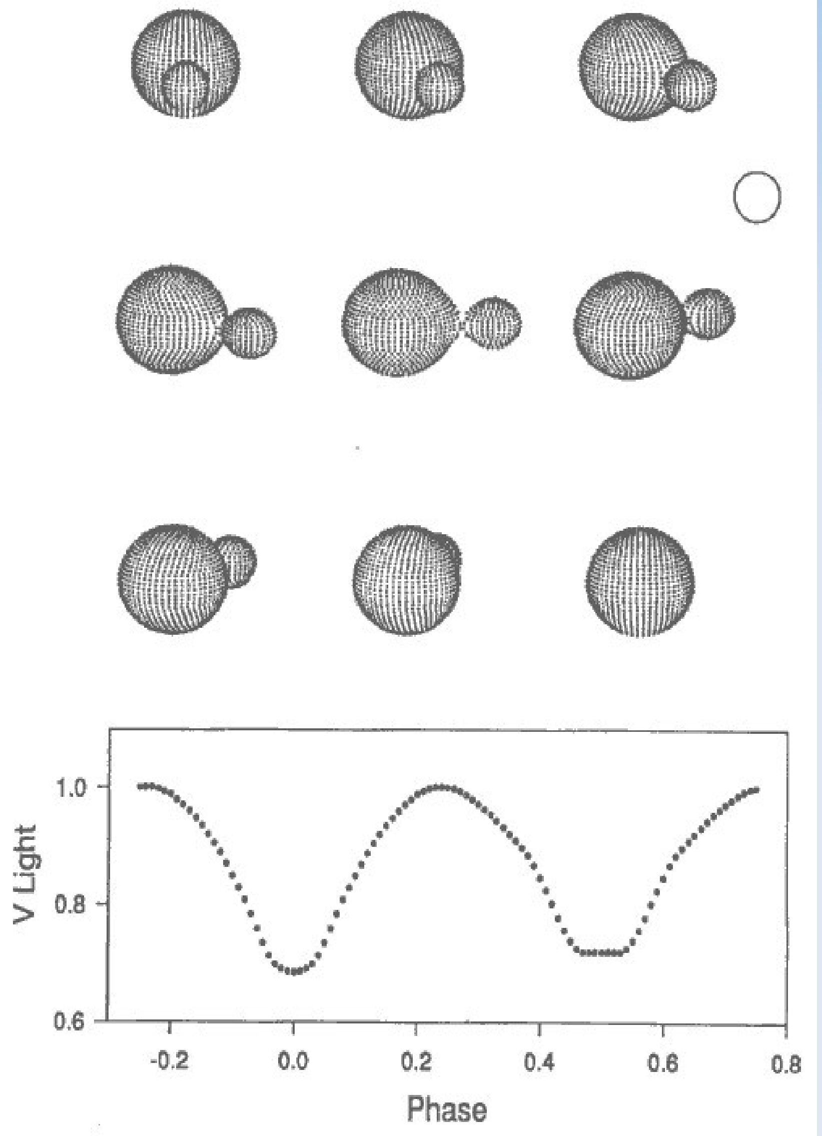


Light-Curve of B. D. + 56°1400.

Muller& Kempf (1903)

W UMa type stars

- Eclipsing binary variable stars, contact binaries
- Very common, 1% of stars belong to this type
- Components of spectral type A/F and periods of P=0.4-0.8 days or types G/K and P=0.2-0.4 d
- Lightcurves show ellipsoidal variations rather than eclipses



RR Cen

Concept of common envelope

The system has mass ratio M1/M2=3.

The black line is the Roche equipotential surface. The dashed line is the rotation axis.

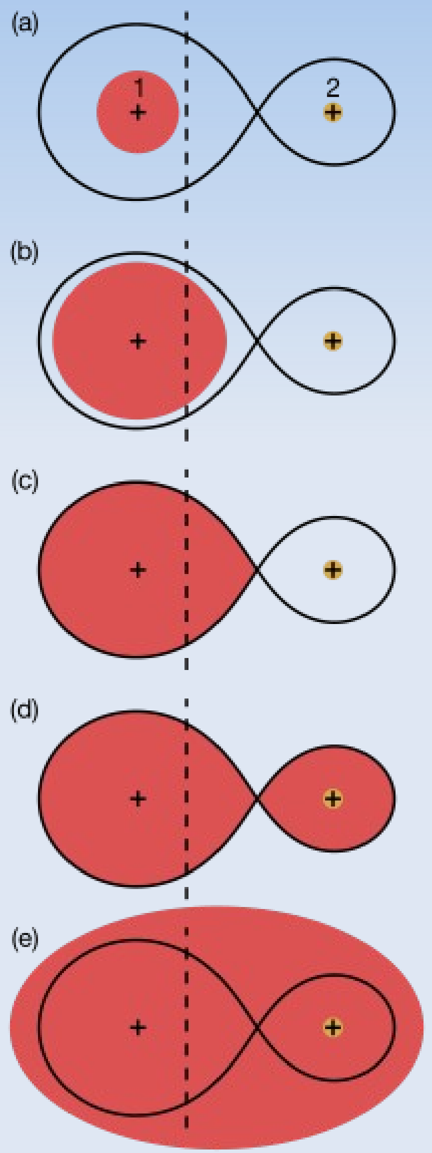
(a) Both stars lie within their Roche lobes, star 1 on the left (mass M1 in red) and star 2 on the right (mass M2 in orange).

(b) Star 1 has grown to nearly fill its Roche lobe.

(c) Star 1 has grown to overfill its Roche lobe and transfer mass to star 2: Roche lobe overflow.

(d) Transferred too fast to be accreted, matter has built up around star 2.

(e) A common envelope, represented schematically by an ellipse, has formed.



Common envelope

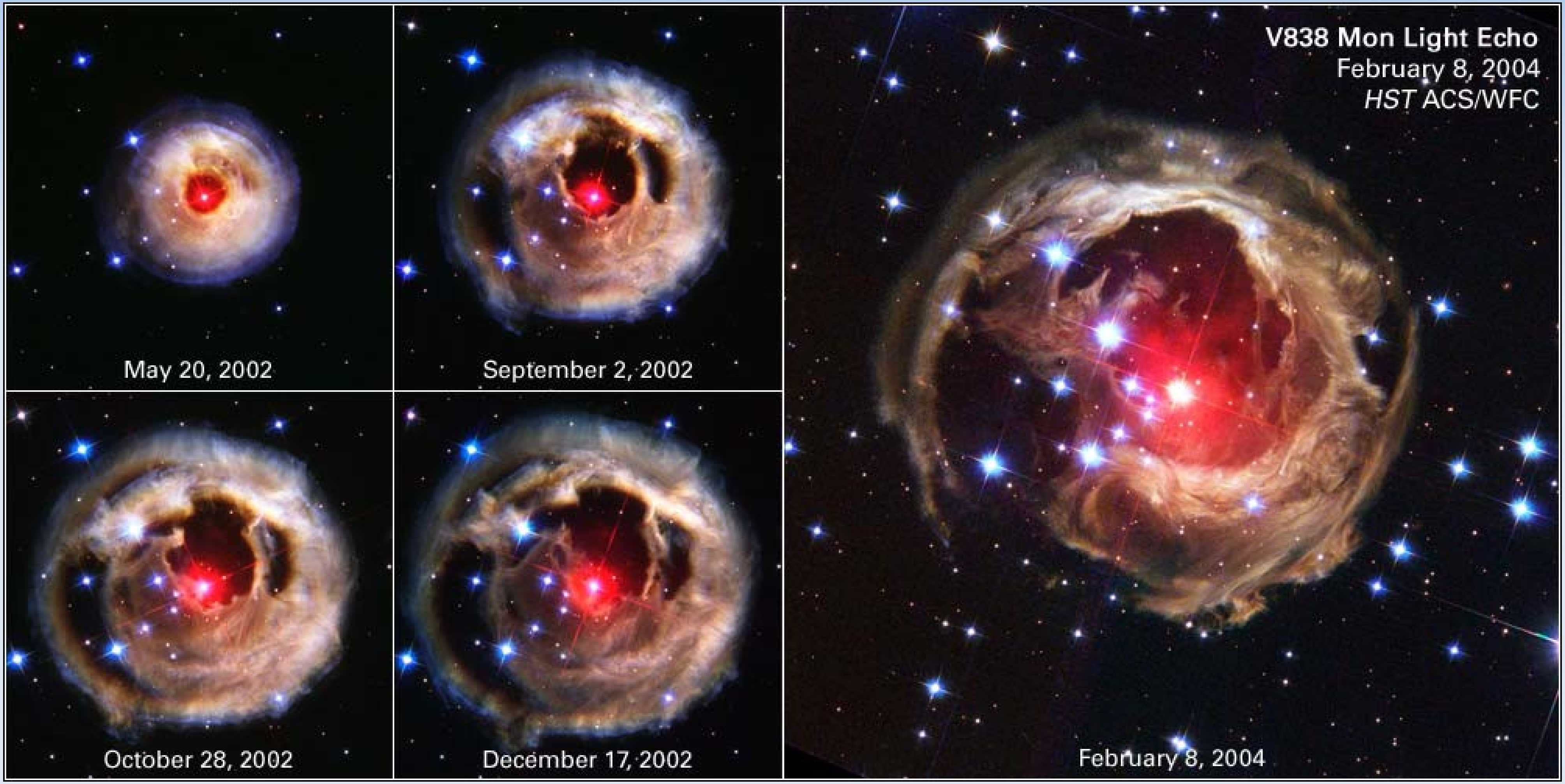
- The mass loss through the L_2 point takes away the angular momentum from the binary system
- The orbit shrinks and expanding envelopes of evolved stars for a common envelope
- Envelope cannot keep the rigid rotation. Differential rotation instead.
- Two cores orbit inside. They transfer angular momentum to the unstable envelope, which finally blows out
- Remaining planetary nebula with a tight binary

V 838 Mon

- Common envelope phase is very difficult to observe.
- We see the binaries either before or after CE phase
- V838 Mon is one of a few candidates.
- They are called a ”luminous red nova”

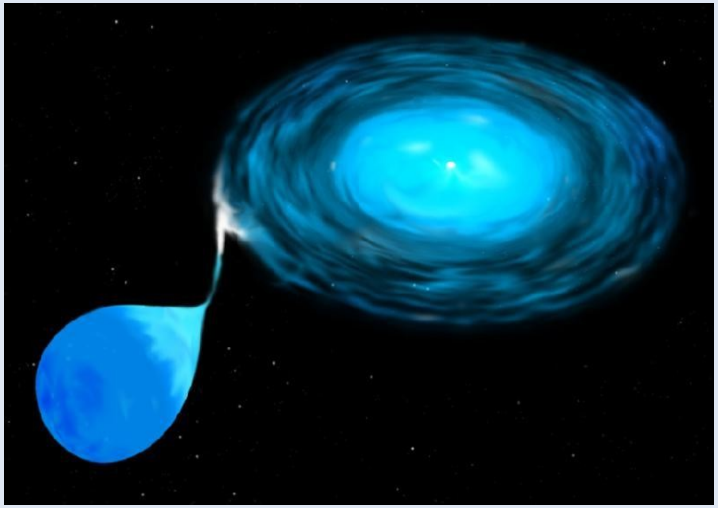
V838 Mon seen by HST

Light echo expansion V 838 Mon



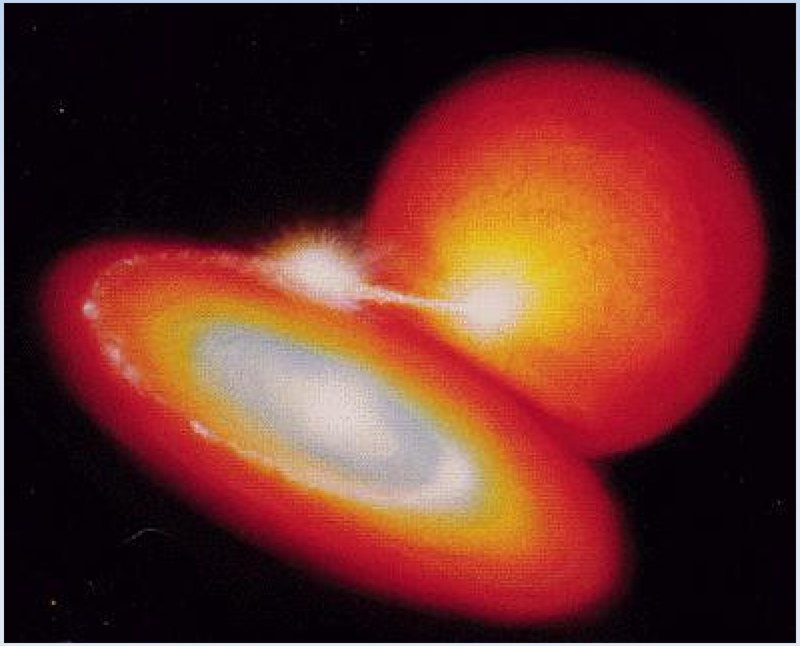
After common envelope phase

- The more evolved star becomes a white dwarf inside the common envelope.
- The companion still in the red giant phase, looses mass. We observe a cataclysmic variable – short period semi-detached system.



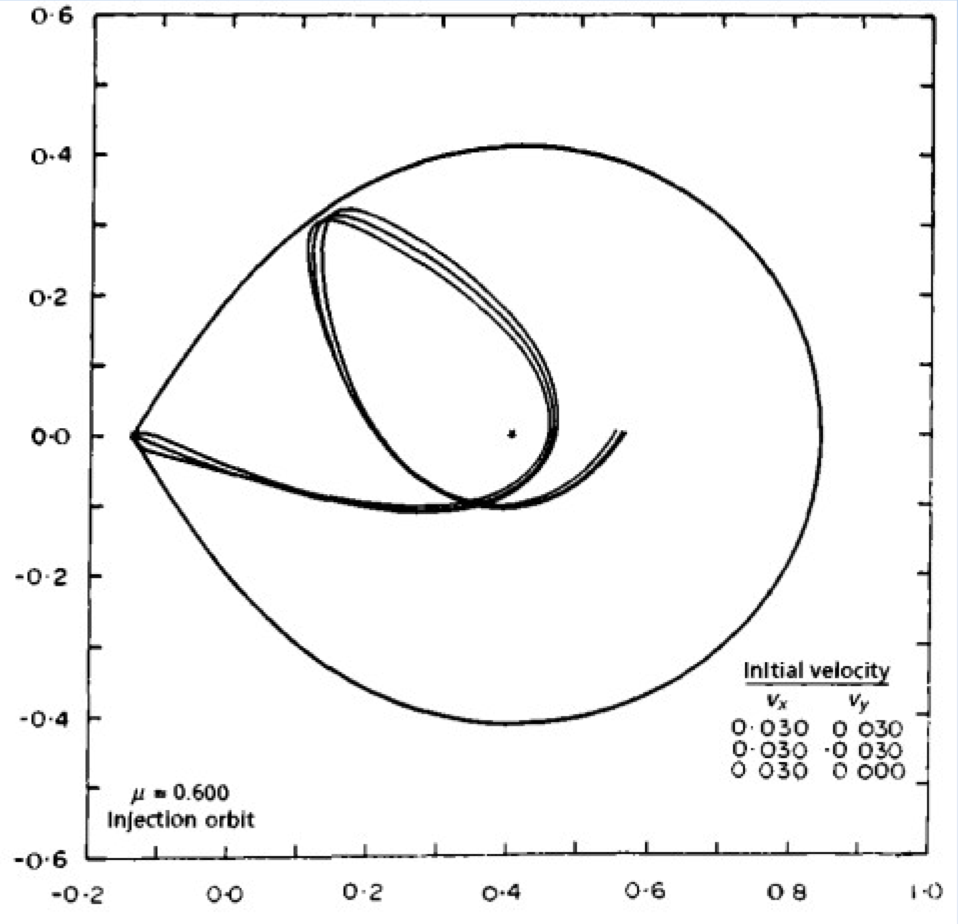
Semi-detached system

- Mass outflowing from the L1 point has too much angular momentum to fall radially
- Accretion disk forms around the primary star and spirals in
- Magnetosphere can halt accretion and prevent disk formation



Flow through the nozzle

- At L_1, velocity of the flow is equal to the sound speed of gas, governed by the temperature of the secondary star's surface layers
- This is $c_s << v_{orb}$
- Stream trajectory follows single particles ejected from L_1



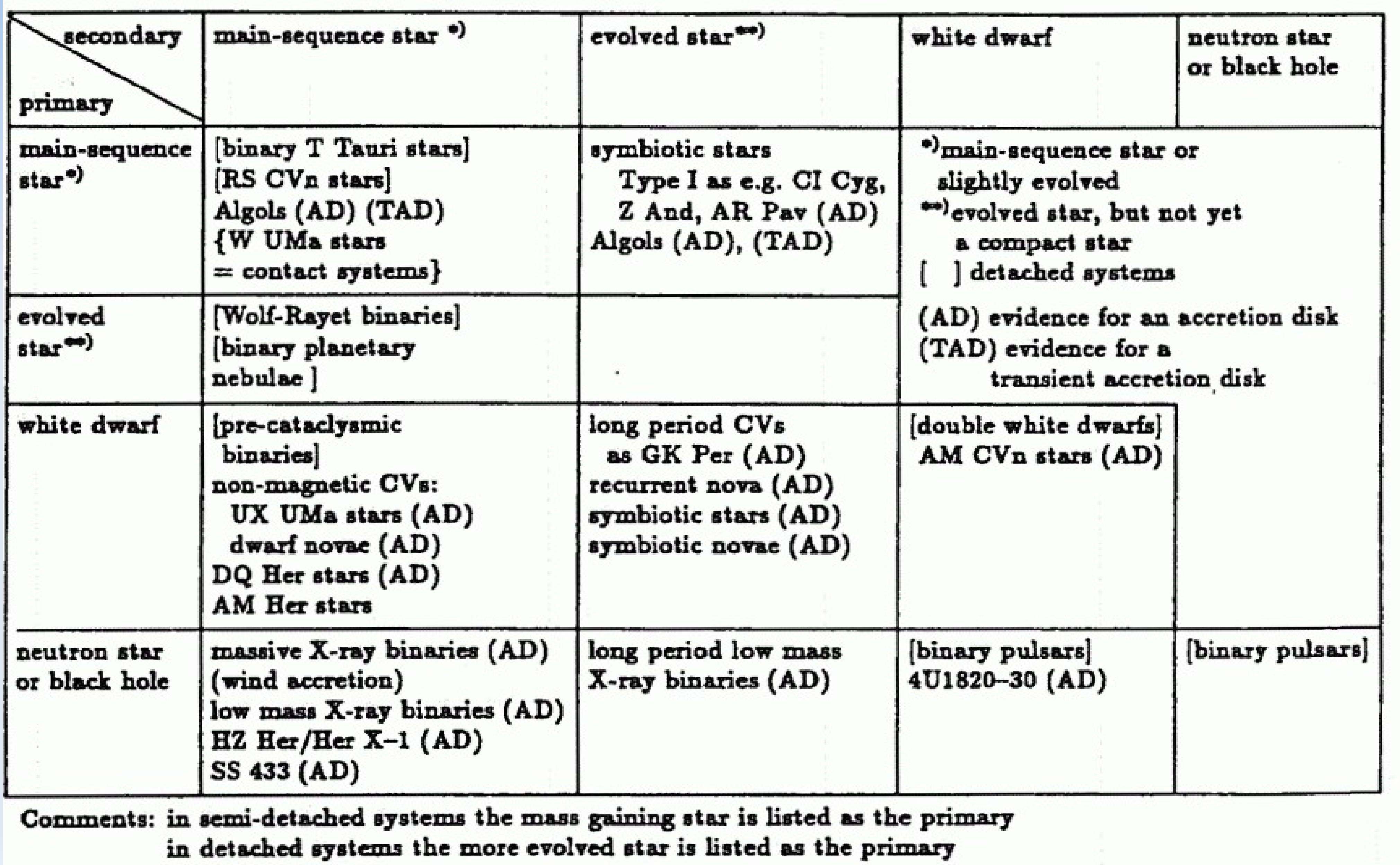
Accretion disk

- If the accreting star is large, the stream will not circularize on its orbit but first hit the star directly
- If the accretor is a compact star (white dwarf, neutron star, black hole) then the stream will circularize at $R_{circ} > R_{star}$
- Accretion disk forms inside this circularization radius

$$l = \sqrt{G\,M\,R_c}$$

- For $M=10\,M_{sun}$ and $l=8\times10^{17}$ cm^2/s, we have $R_c=5\ 10^8$ cm

Close binary systems



secondary / primary	main-sequence star *)	evolved star**)	white dwarf	neutron star or black hole
main-sequence star*)	[binary T Tauri stars] [RS CVn stars] Algols (AD) (TAD) {W UMa stars = contact systems}	symbiotic stars Type I as e.g. CI Cyg, Z And, AR Pav (AD) Algols (AD), (TAD)	*)main-sequence star or slightly evolved **)evolved star, but not yet a compact star [ ] detached systems	
evolved star**)	[Wolf-Rayet binaries] [binary planetary nebulae]		(AD) evidence for an accretion disk (TAD) evidence for a transient accretion disk	
white dwarf	[pre-cataclysmic binaries] non-magnetic CVs: UX UMa stars (AD) dwarf novae (AD) DQ Her stars (AD) AM Her stars	long period CVs as GK Per (AD) recurrent nova (AD) symbiotic stars (AD) symbiotic novae (AD)	[double white dwarfs] AM CVn stars (AD)	
neutron star or black hole	massive X-ray binaries (AD) (wind accretion) low mass X-ray binaries (AD) HZ Her/Her X–1 (AD) SS 433 (AD)	long period low mass X-ray binaries (AD)	[binary pulsars] 4U1820–30 (AD)	[binary pulsars]

Comments: in semi-detached systems the mass gaining star is listed as the primary
in detached systems the more evolved star is listed as the primary

Next week

- More about evolution of massive and low mass X-ray binaries. Disk accretion

Suggested literature:

- Izzard R., et al., 2012, IAU proc. 283, 95
- Kopal, Z. ”Close binary systems”, Wiley NY, 1959