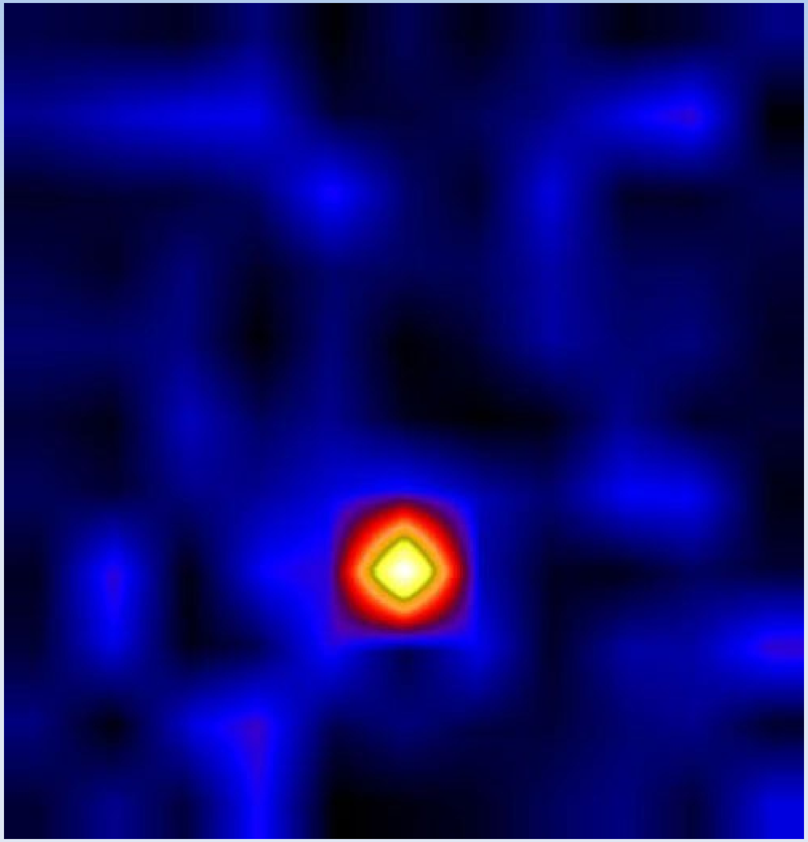
Compact Stars



Lecture 12

Today: Gravitational Waves

- Predicted by the Theory of General Relativity.
- Accelerating objects generate changes in the spacetime curvature, which propagate outwards at the speed of light in a wave-like manner.
- These propagating ripples are known as gravitational waves.
- Observer will find spacetime distorted: strain is displacement between particles in the detector relative to a reference length.

Gravitational waves

- Waves are motions of background medium (water, or spacetime).
- The GWs wavelength is very small compared to radius of curvature of the spacetime through which they propagate.
- Strength of background gravitational field is related to tidal force, $F_{tid} \sim M/L^3$, where M is mass of the source of gravity and L is characteristic size.
- Radius of curvature scales as $F_{tid}^{(-1/2)}$. In Solar system, R~ 0.1 lyr.

Properties of gravitational radiation

- In weak field, linearized metric tensor has the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Gravitational waves are described by two dimensionless amplitudes, h_+ and h_x.
- They are functions only of t-z/c, if the wave propagates in z-direction.
- The tensor h^{TT} is a symmetric spatial tensor, traceless and transverse to direction of GW propagation (no z-component)

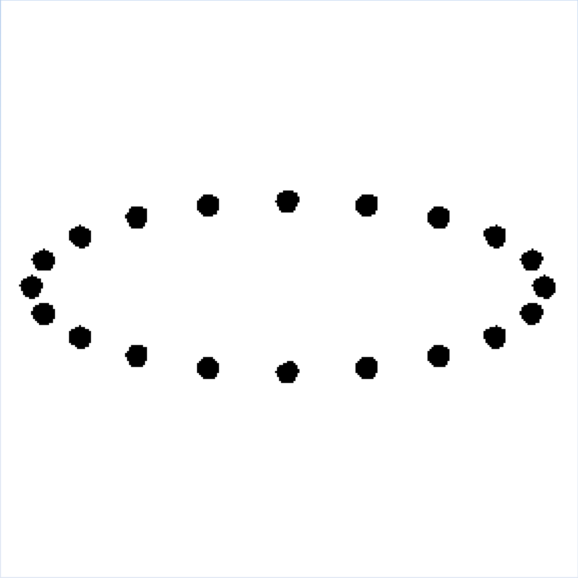
Space-time metric

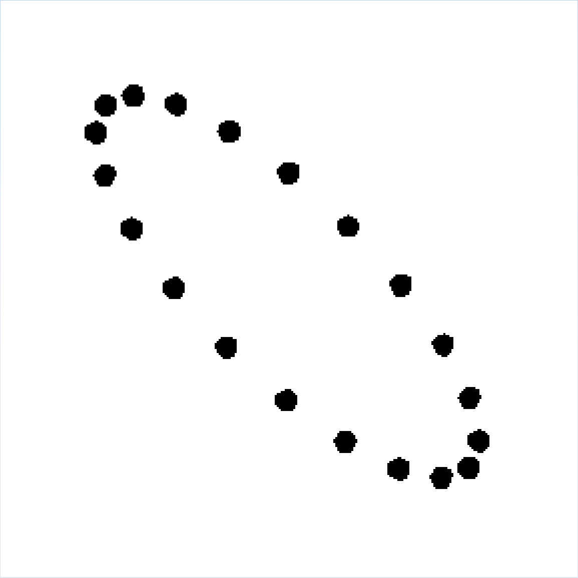
- $[g_{\mu\nu}]=[\eta_{\mu\nu}]+[h_{\mu\nu}]=$

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix} \quad + \quad \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \quad \cos(\omega t-kz)$$

- $ds^2=g_{\mu\nu}dx_\mu dx_\nu=c^2dt^2-[1-h_+\cos(\omega t-kz)]dx^2+2h_\times\cos(\omega t-kz)dxdy-[1+h_+\cos(\omega t-kz)]dy^2-dz^2=c^2dt^2-dl^2.$

Effect on ring of particles





- The effects of a passing gravitational wave can be visualized by imagining a perfectly flat region of spacetime with a group of motionless test particles lying in a plane, e.g., the surface of the screen. As a gravitational wave passes through the particles along a line perpendicular to the plane, the particles will follow the distortion in spacetime, oscillating in a "cruciform" manner (in x-y direction for the ”+” ,left plot, and diagonally for the ”x”, right)
- The area enclosed by the test particles does not change and there is no motion along the direction of propagation.

Gravitational and electromagnetic waves

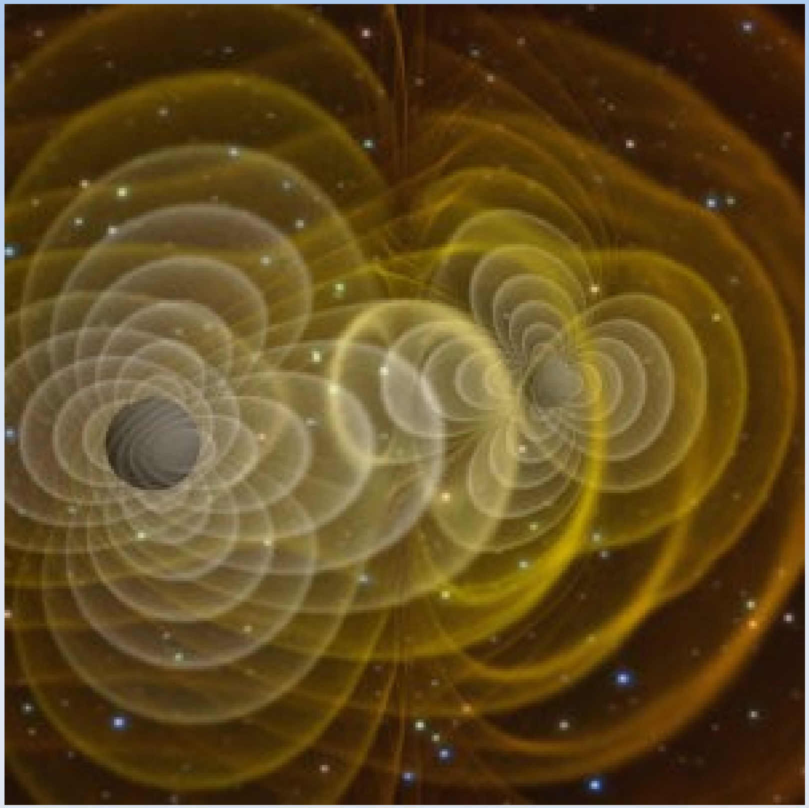
- The h^{TT} is a gravitational radiation potential, analogously to A_i being an electromagnetic radiation vector potential.
- Because h^{TT} is a two-rank tensor, the carriers of GW (gravitons) are spin-two paticles. In contrast, photons (EM radiation carriers) are spin-one particles.
- Two independent polarisation directions of GW are rotated by an angle $\pi/4$ between each other. For EM radiation, it is by angle $\pi/2$.

Detection of GWs

- The detection of gravitational waves is based on the possibilities to measure the tiny relative displacements of test particles due to the interactions with gravitational radiation.
- For a single particle at rest before the arrival of a gravitational wave, it remains at rest. Thus, it is not possible to detect gravitational waves on a single particle.
- What may change is the relative proper distance between two particles, and the effects of gravitational radiation can be detected by measuring it.

Sources of GWs

- In general, any acceleration that is not spherically or cylindrically symmetric will produce a gravitational wave.
  - Rotating star with non-uniform mass distribution
  - Asymmetric supernova ejection
  - Orbiting binaries

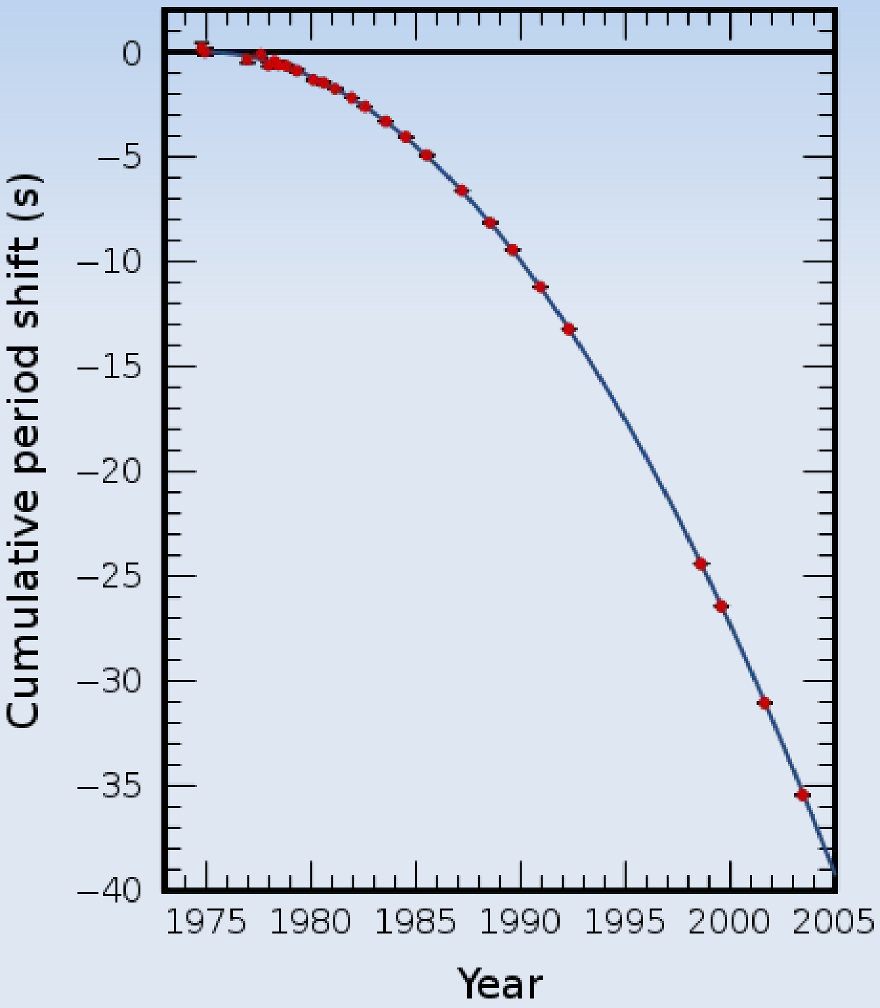


Types of GWs

- Continuous
  - Produced by systems that have a constant, well-defined frequency. Examples: binary sytems of stars or black holes orbiting each other (long before merger), or a single star swiftly rotating about its axis with a large mountain or other irregularity.
- Stochastic
  - Relic from the early evolution of the universe. Likely to be the leftover from the Big Bang, arise from a large number of random, independent events combining to a cosmic gravitational wave background.
- Inspiral
  - Generated during the end-of-life stage of binary systems where the two objects merge into one.

PSR B1913+16

- Pulsar in binary system with another neutron star
- Discovered by R. Hulse and J.Taylor in 1974
- Mass M=1.441
- Spin P=59.02 ms
- Orbital period t=7.75 hrs, decays 76.5 μs/yr
- Time to final inspiral is 300 mln yrs

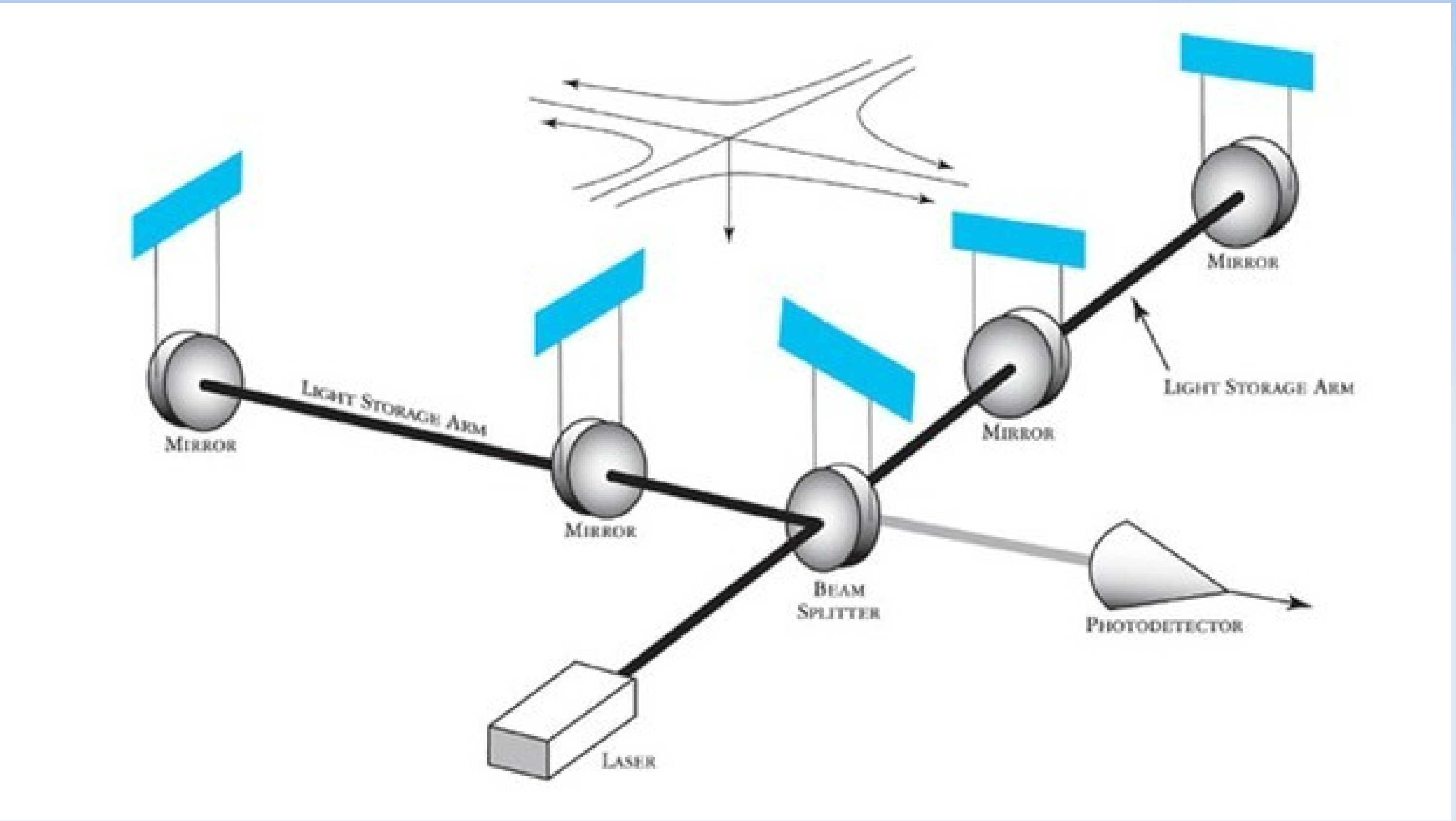


GW Interferometers

- LIGO: "Laser Interferometer Gravitational-wave Observatory". Consists of two instruments, with a very large L-shaped detectors: 4km long arms.
- Other detectors:Virgo, GEO, and KAGRA

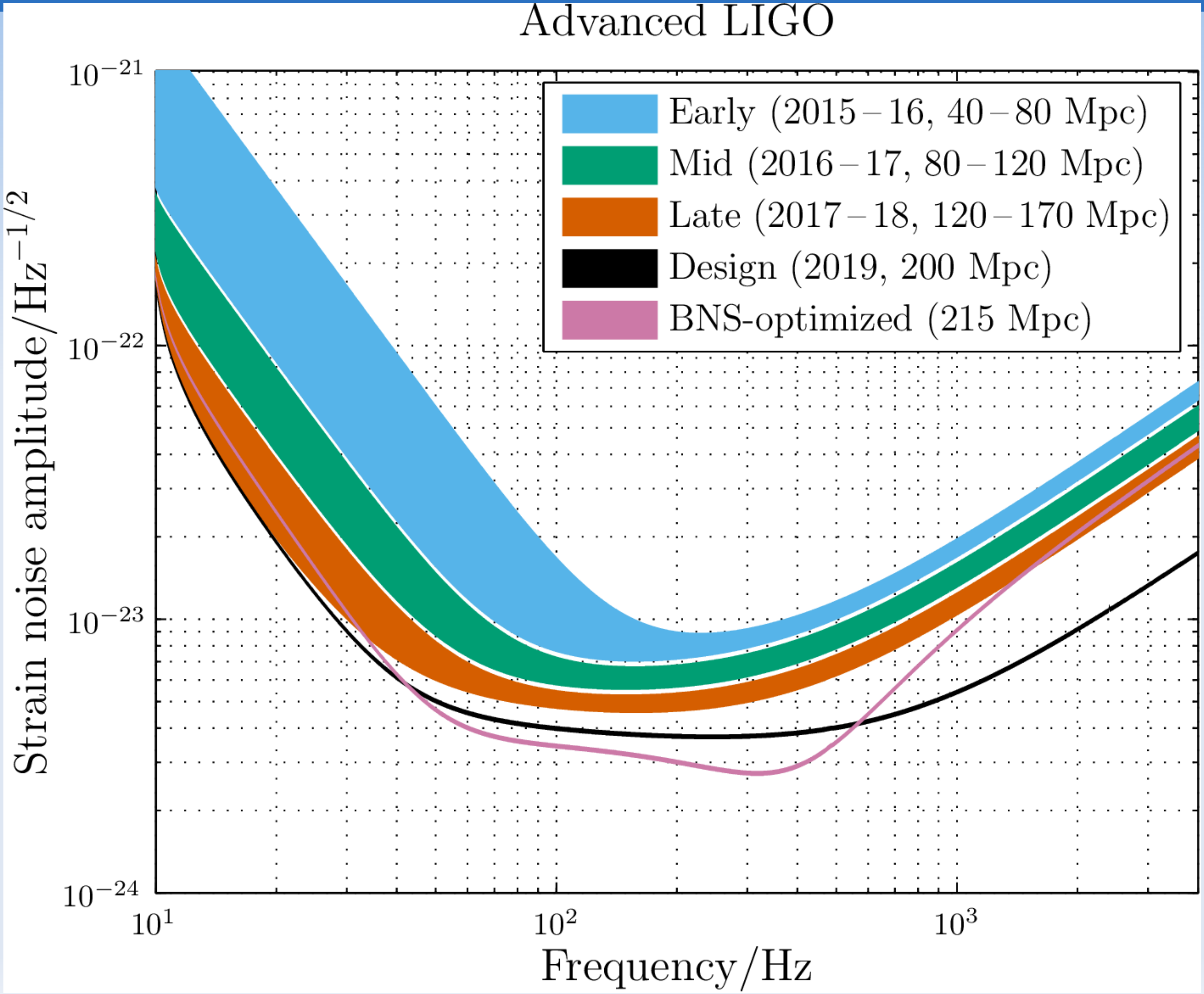
Two main reasons for the wide separation between the interferometers: Local vibrations, and gravitational wave travel time.

GW Interferometry



Gravitational waves cause space itself to stretch in one direction and simultaneously compress in a perpendicular direction. In LIGO, this causes one arm of the interferometer to get longer while the other gets shorter, then vice versa, back and forth as long as the wave is passing. The technical term for this motion is "Differential Arm" motion, or differential displacement, since the arms are simultaneously changing lengths in opposing ways.

Detector sensitivity



Inspiral-Merger-Ringdown

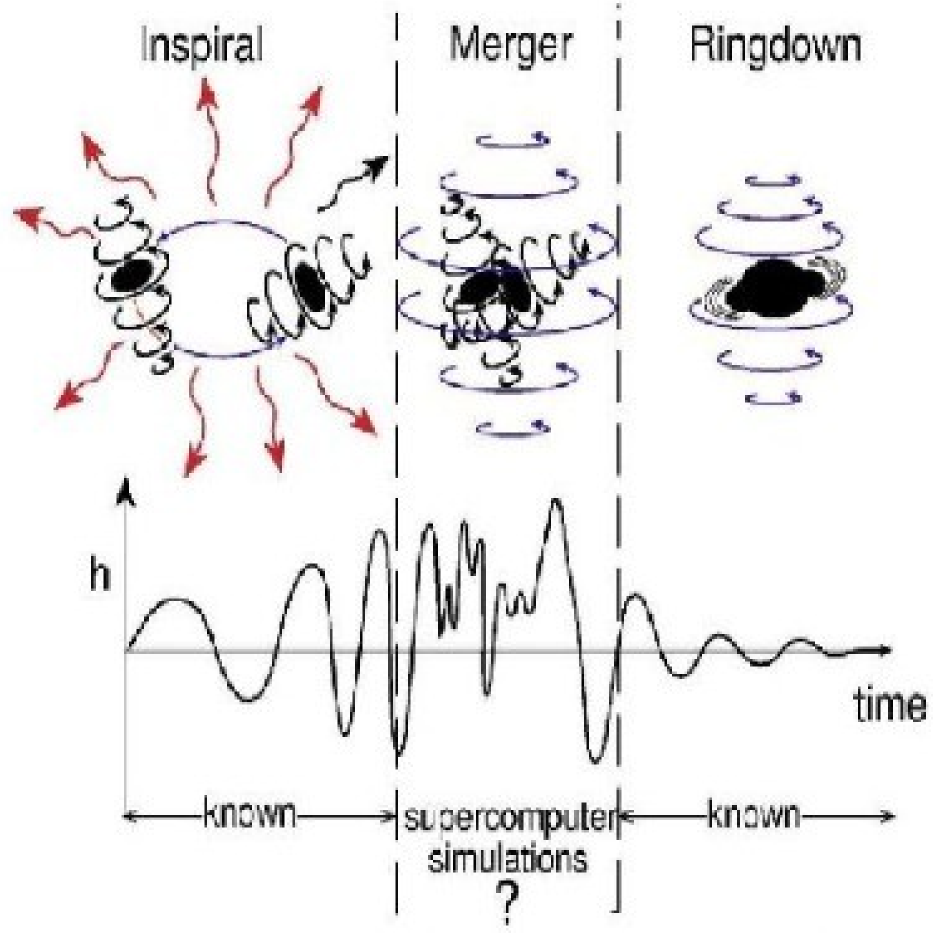


Fig. 1. The three phases of black hole merger (courtesy Kip Thorne).

Two body problem in GR

- Two-body problem in Newtonian physics is solved, e.g. planets motion along the elipse
- Two body problem in General Relativity is unsolved analytically.
- Numerical relativity provides tools for discretizing field equations and evolution schemes
- Simulations allow to study merger process

Inspiral-Merger-Ringdown

- The test particle limit applies to describe motion of perihelion precession or chaotic orbits
- For equal-mass mergers, the first, inspiral phase, is treated with the post-Netownian expansion series, accounting for self-gravity. Slow motion ($v<<c$) is assumed and weak field
- Ringdown of the final product is described by perturbation theory

Stable numerical schemes for BBH mergers

- Must choose the coordinates and systems of variables that do not develop pathologies when the system is evolved
- The system of hyperbolic and elliptic equations is solved for constrained evolution
- The scheme must deal with geometric singularities in black hole spacetimes

Numerical relativity

- Two main schemes are used
  - Generalized harmonic coordinates with constrained damping (Pretorius 2005)
  - ”Moving punctures’ method with conformal connection functions to the standard equations (Shibata et al. 2006). The method is called BSSN (from the names of authors)

Each equation contains hundreds terms, requiring on several thousand oating point operations per grid point with any evolution scheme

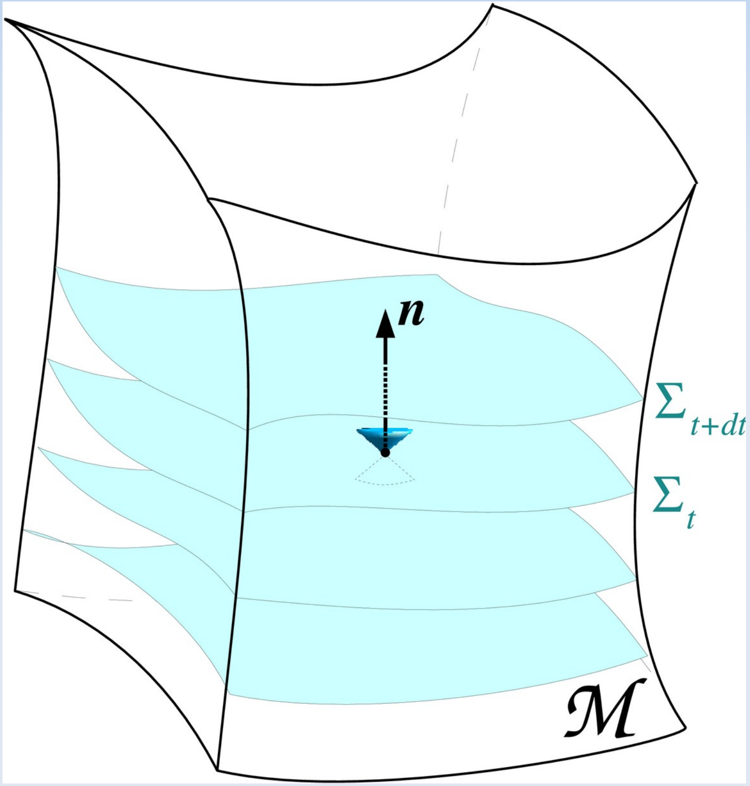
- Geometric singularities inside black holes are treated with ’excision’ or special criteria for the lapse and shift functions (’moving punctures’)
- Several orders of magnitude of relevant physical length need to be resolved. This is done with the Adaptive Mesh technique

3+1 decomposition

Numerical implementation of the 3 + 1 split of Einstein equations, for solving the Cauchy initial value problem using the BSSN method.

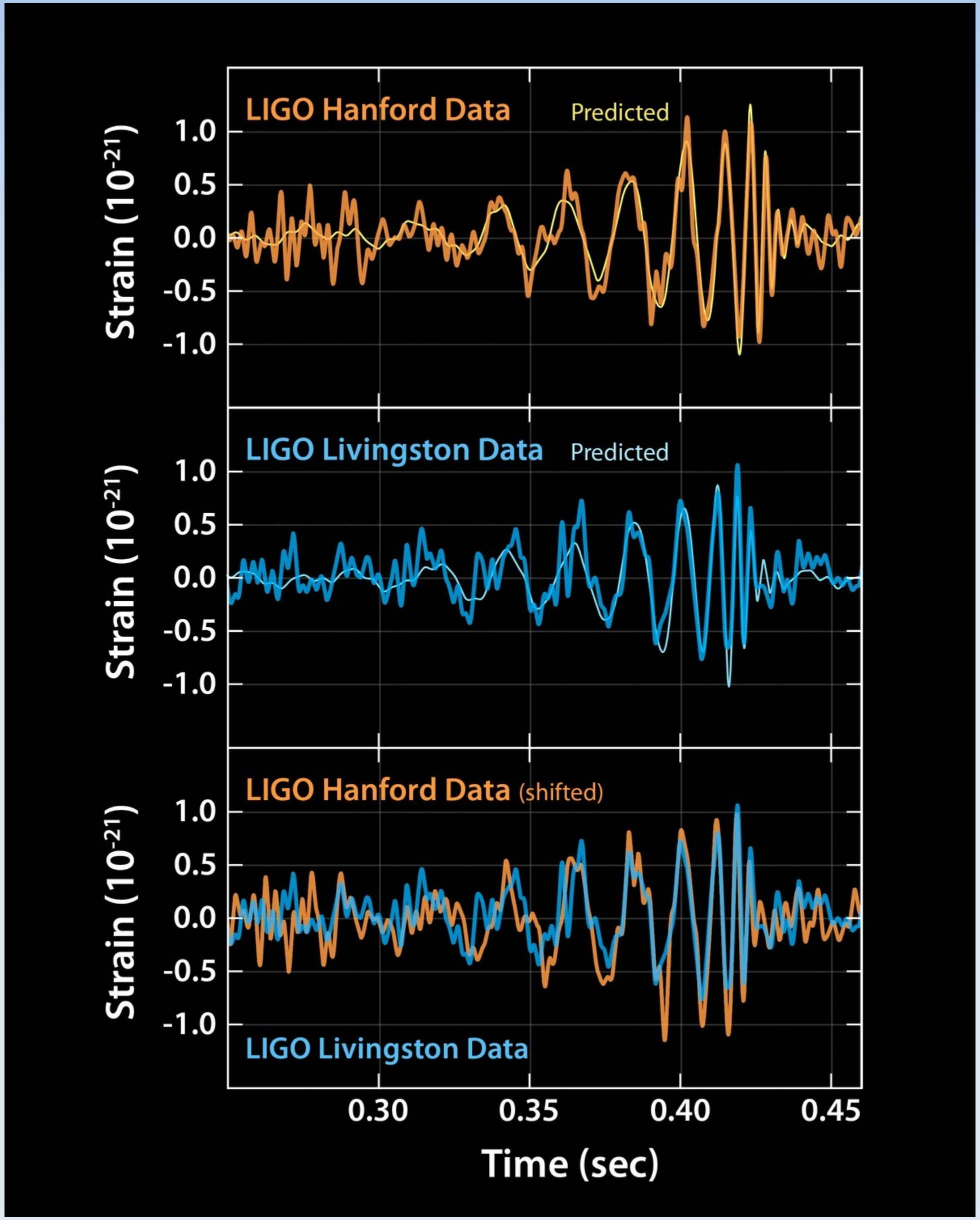
Formulation assumes that spacetime is foliated into a family of 3-dimensional spacelike hypersurfaces. Proposed by Arnowitt, Deser and Misner (ADM formulation).

Fundamental variables in the BBH evolution are spatial metric γ_{ij} and extrinsic curvature tensor K_{ij}



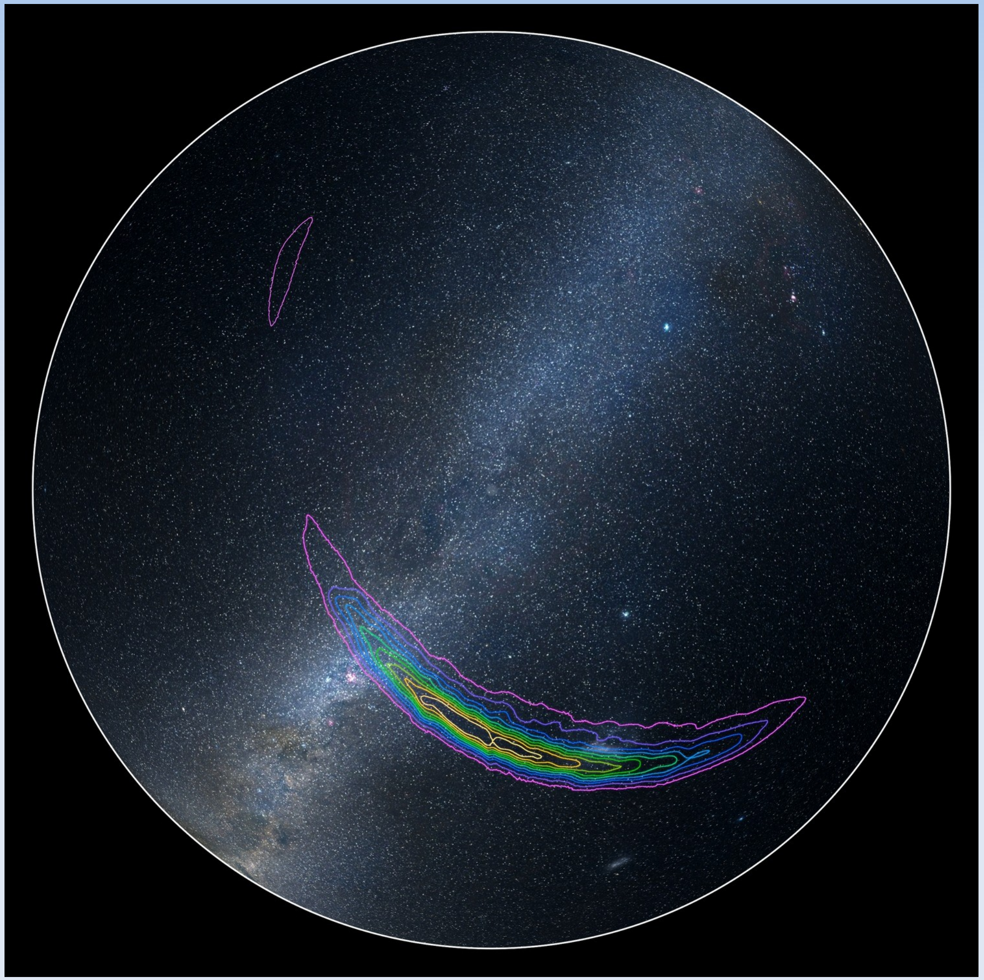
First BBH merger detection

- Source GW 150914:
- On February 11, 2016, the LIGO and Virgo Collaborations announced the first confirmed observation of gravitational waves from colliding black holes.
- The gravitational wave signals were observed by the LIGO's twin observatories on September 14, 2015.



Source localisation

- The approximate location of GW150914 on a sky map of the southern hemisphere.
- The colored lines represent different probabilities for where the signal originated: from a 90 percent confidence level; to a 10 percent confidence level. T
- The gravitational waves were produced by a pair of merging black holes located 1.3 billion light-years away.
- Estimated masses of black holes for this event were about 29 and 36 times the mass of the Sun



Binary black holes

Binary black holes may exist in the nuclei of merged galaxies.

Galaxy mergers are classified to:

- minor mergers (one of the galaxies is much smaller than the other) and major mergers (similar sizes)

- wet mergers (black holes merge in a gas rich environment) and dry mergers (galaxies are gas-poor)

”Mice galaxies”: two galaxies in a process of merger

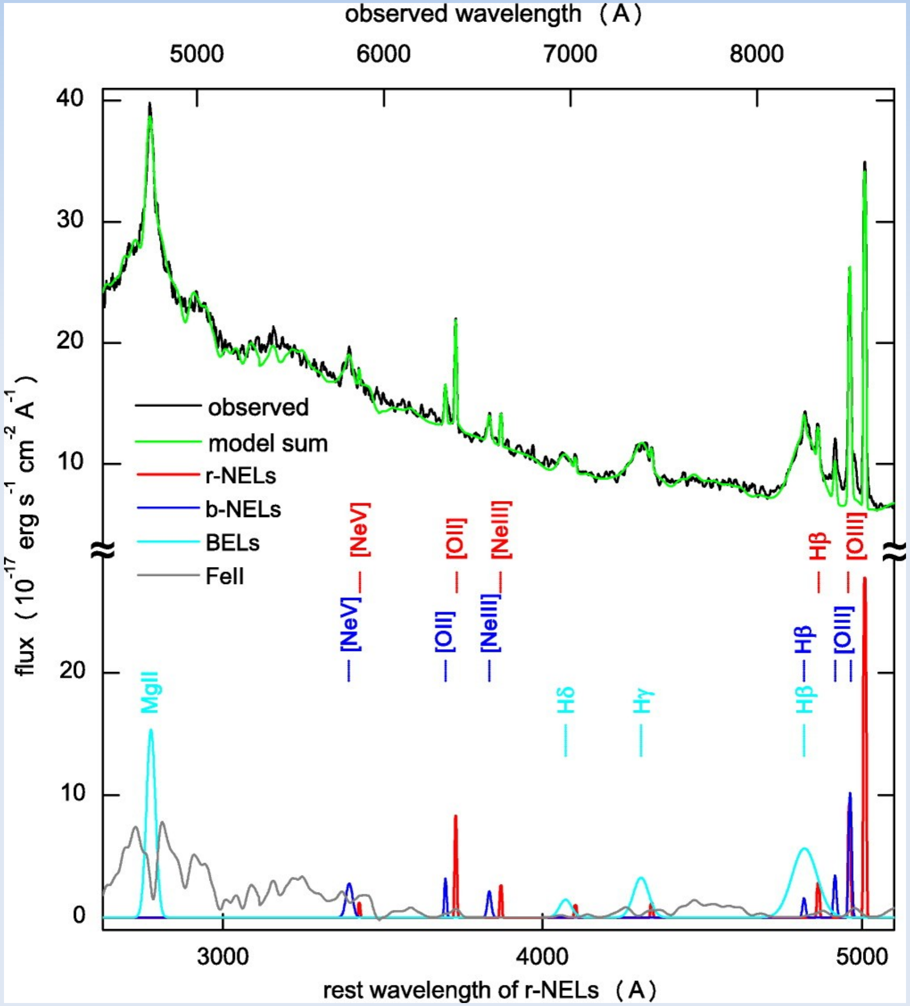
Gravitational recoil of black holes

- SDSS J092712.65+294344.0 as the best candidate to date for a recoiling supermassive black hole (SMBH)
- Observations of emission lines from the source suggest that this might have been a gravitationally recoiled AGN nucleus (Komossa et al. 2008).
- Broad Balmer and broad high-ionization forbidden lines which are blueshifted by 2650 km s-1 relative to the set of narrow emission lines. This observation is most naturally explained if the SMBH was ejected from the core of the galaxy, carrying with it the broad-line gas while leaving behind the bulk of the narrow-line gas.

Spectrum of SDSS J092712.65+294344.0

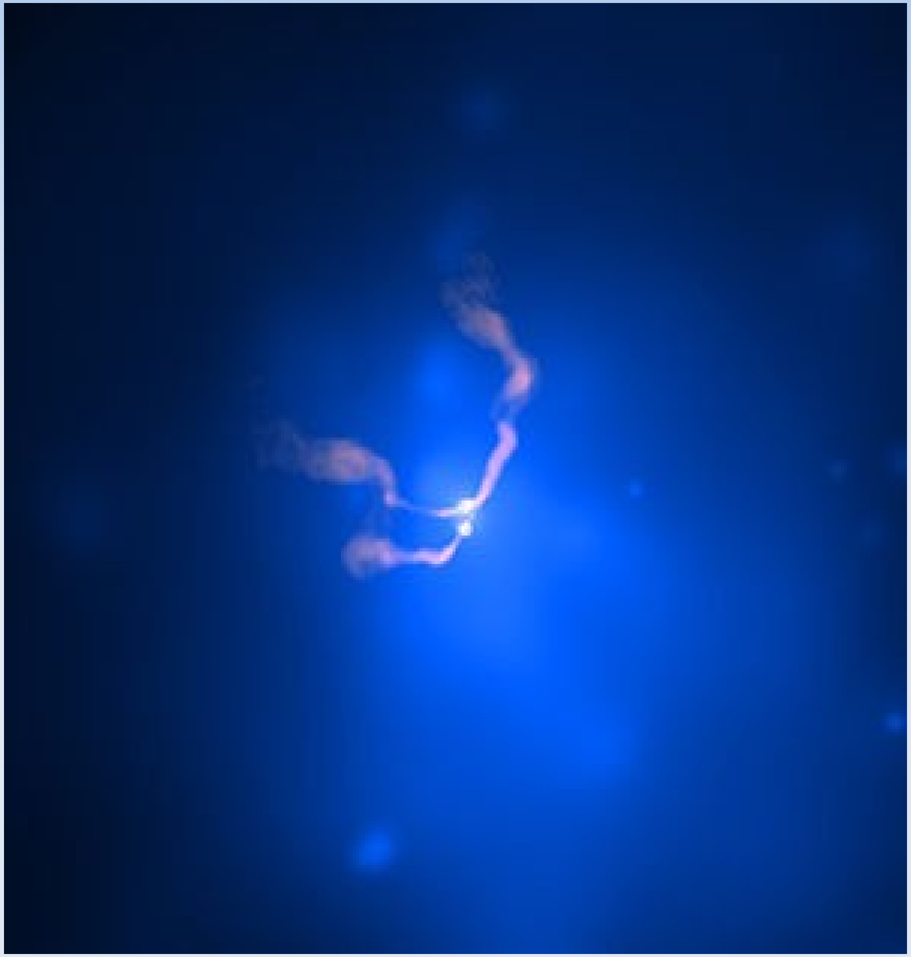
Two sets of emission lines separated by a velocity of 2650 km/s.

Red: Red set of narrow emission lines (r-NELs). Blue and light blue: Blue set of emission lines (b-NELs and BELs, respectively). Gray: Fe II spectrum.



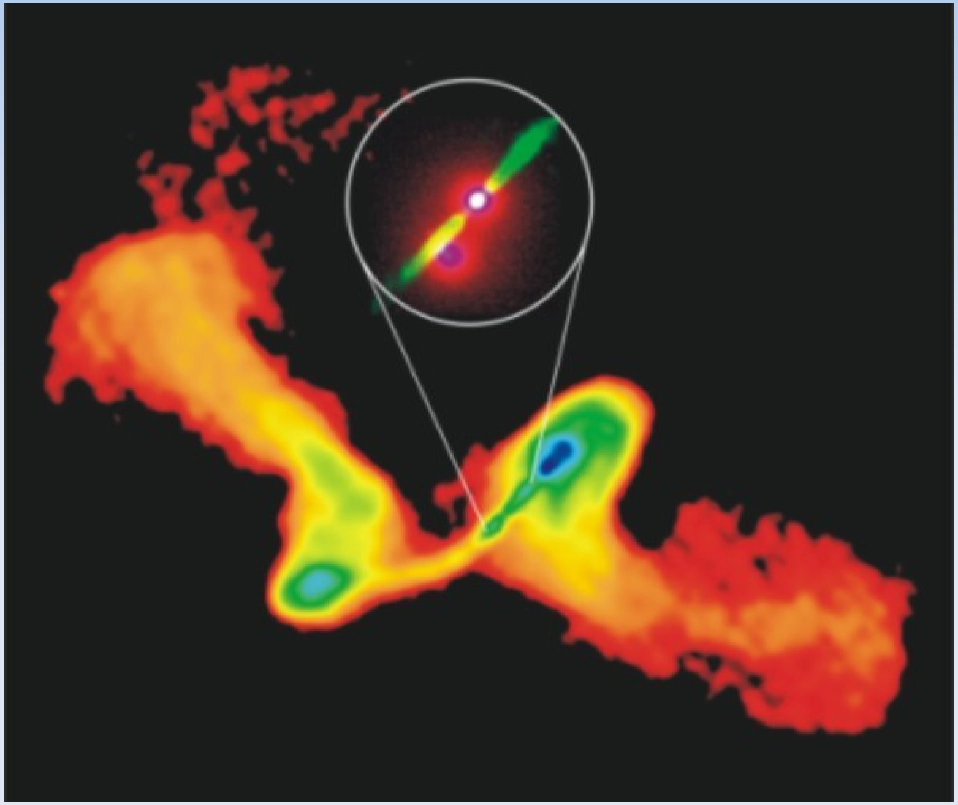
NGC 1128

- Abel 400 is a galaxy cluster
- Two merging galaxies there are seen as object NGC 1128, and in radio band identified as 3C 75
- Four jets indicate presence of two accreting black holes
- X-ray image (blue) and radio image (pink color)



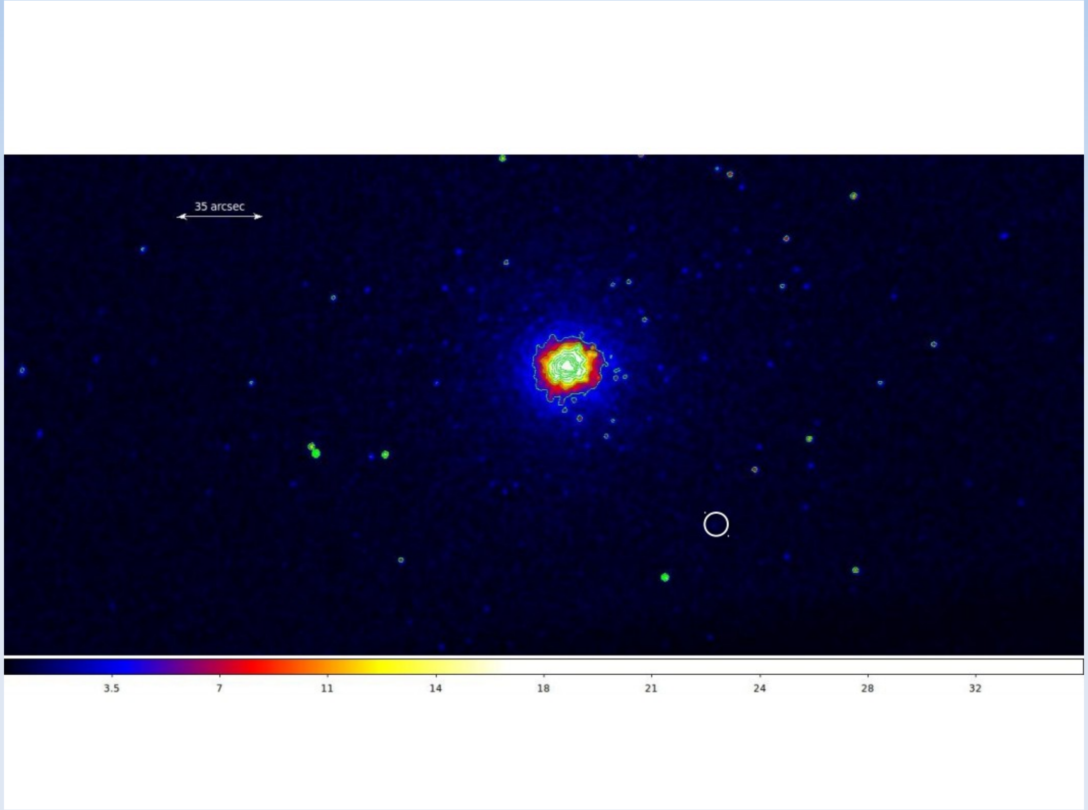
NGC 326

- ”X-shaped” morphology seen on radio maps
- Kinked jet, two fainter lobes oriented with an angle w.r.t brighter ones
- Possible effect of the past merger



M60-UCD1

- Ultra-compact dwarf galaxy in the Virgo cluster
- Contains a supermassive black hole with mass 2.1e7 Msun.
- Black hole contains about 20% of mass of this galaxy
- Its relative velocity w.r.t center to the galaxy is about 240 km/s



- X-ray image: M. Kunert-Bajraszewska
- Orphan black hole ejected from the host?

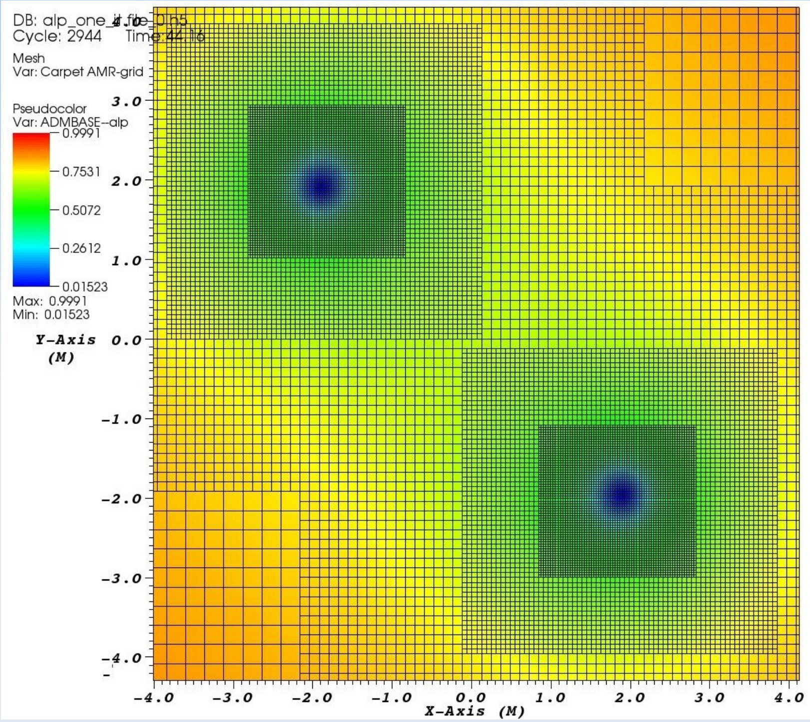
Gravitational recoil

For non-spinning, unequal mass components, the kick velocity of remnant black hole occurs, due to asymmetric beaming of radiation

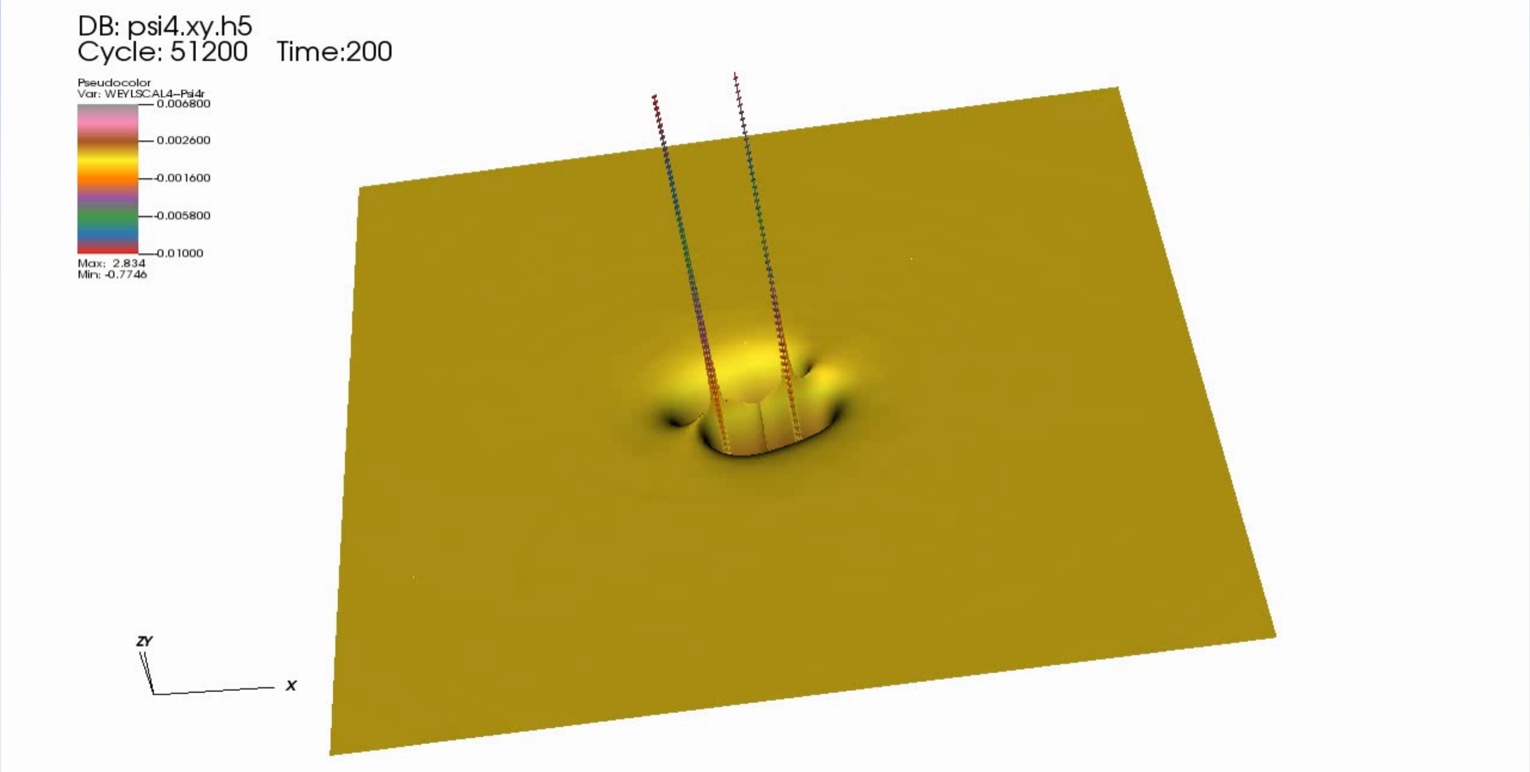
- Recoil speed computed to be up to 175 km/s for non-spinning, unequal mass components
- Typical values for spinning black holes of 100 s km/s, but can be as large as 4000 km/s for equal mass black holes with spins vectors anti-aligned and in the orbital plane

Einstein Toolkit package

- Einstein Toolkit computational framework: a family of codes for use in relativistic astrophysics based on nite dierence computation on a gridded mesh (Loeffler et al. 2012).
- The Toolkit is supported by a distributed model, combining core support of software, tools, and documentation in its SVN and GIT repository with partnerships of developers
- The latest version named Tesla (16th) was released on Feb 2018
- The code documentation can be found on http://www.einsteintoolkit.org

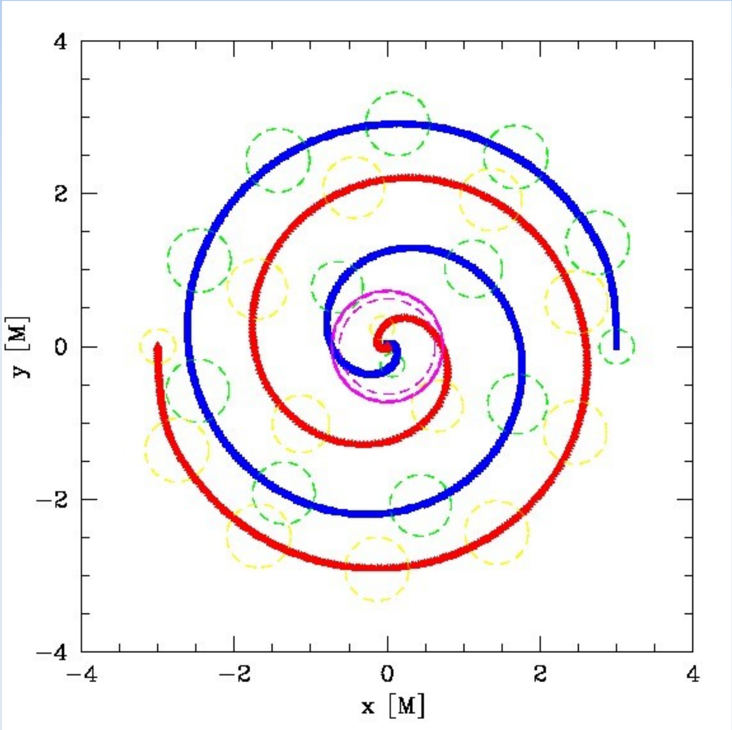


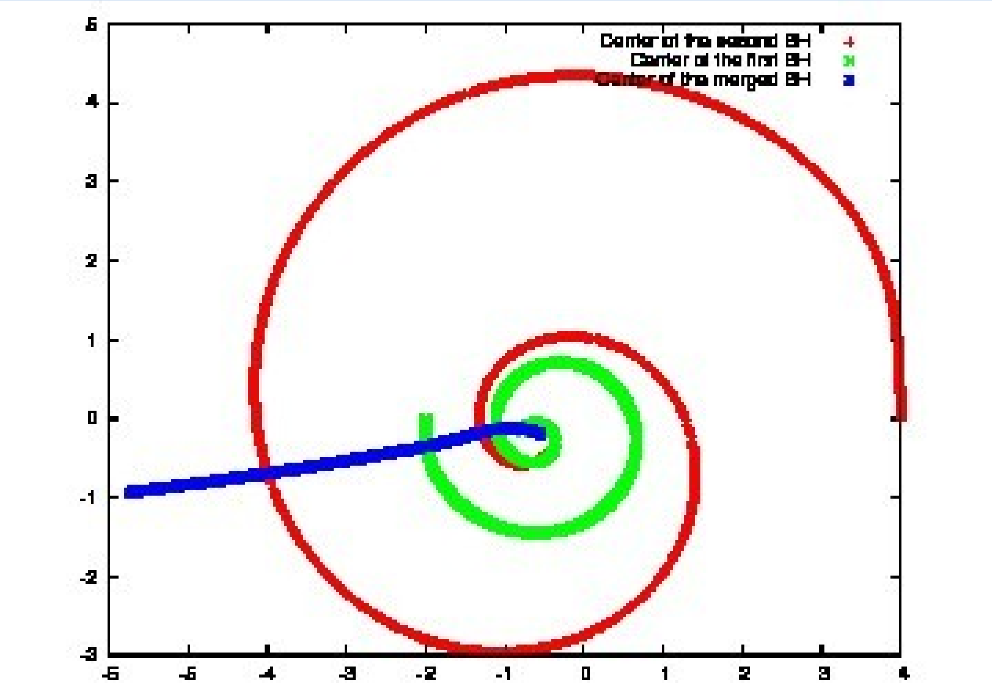
Gravitational wave visualisation



Visualisation: P. Sukova (2017)

Sample orbits





Orbits of the inspiraling black holes, for equal masses (left) and mass ratio $M_1/M_2=3$ (right). Apparent horizons are marked with dashed circles. Recoil of merger product is marked with blue line on the right.

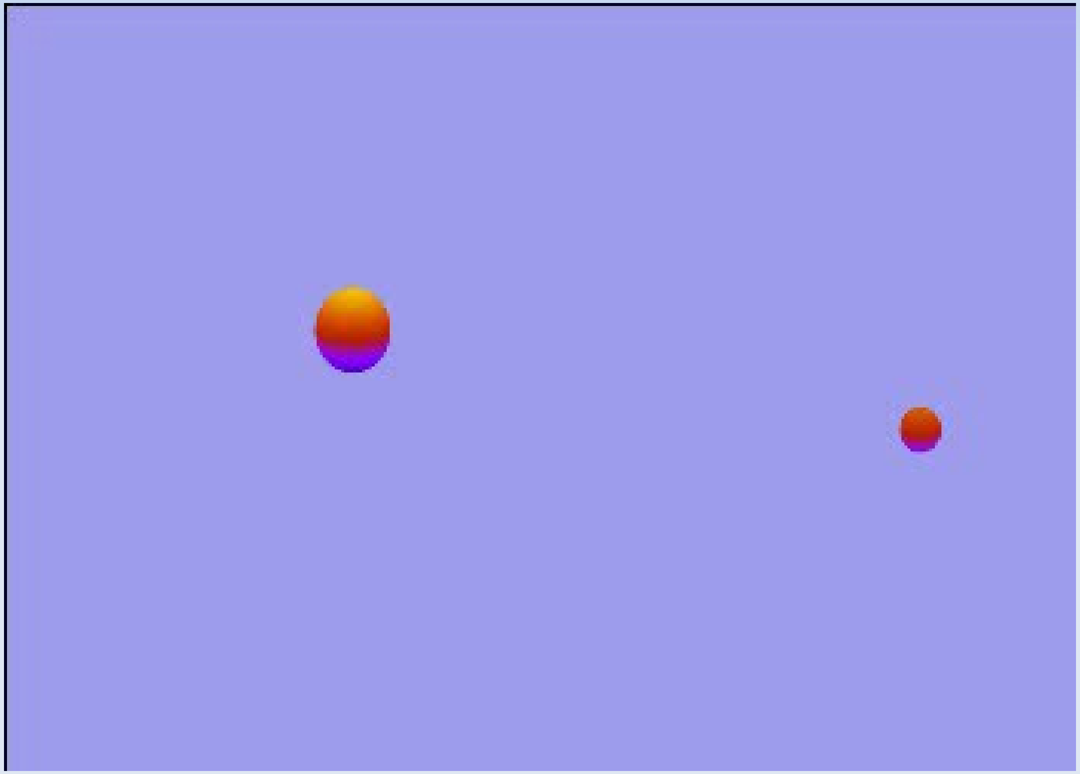
Apparent horizons

The apparent horizons are localized around the components of the BH system in each slice of time during the evolution, and around final merged black hole after it forms.

The AH shape
is given by a function r = h(angle), found by solving a nonlinear elliptic PDE in h on S2

The proper integrals over the isolated horizon are calculated to extract the values of mass and spin of the merged black hole (treated 'quasilocally'; Dreyer et al. 2003).

Apparent horizons of two merging black holes



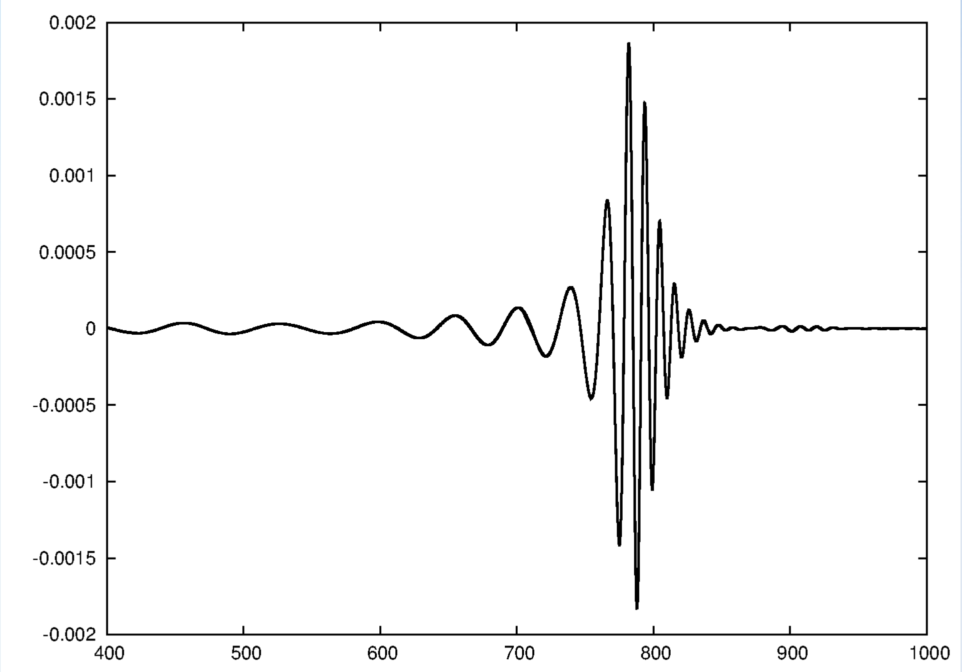
Gravitational wave analysis

Calculation of the exact value of the recoil speed requires the evaluation of the momentum carried away by the gravitational radiation during the merger.

To calculate total momentum carried by radiation we have followed algorithm described by Alcubierre (2008).

We use the formula for dP/dt in terms of coecients Alm of multipole expansion of the Weyl scalar Psi_4 .

The coefficients A^{lm} are computed on the sphere of a given radius, and with l ranging from 2 to 4.



Psi-4 in time of the simulation

Exemplary gravitational wave signal: the real part of the l = 2, m = 2 multipole component of the Weyl scalar Psi-4 extracted at the sphere of radius 42M

Recoil velocities

The velocity of the final BH depends on spins and masses of the components.

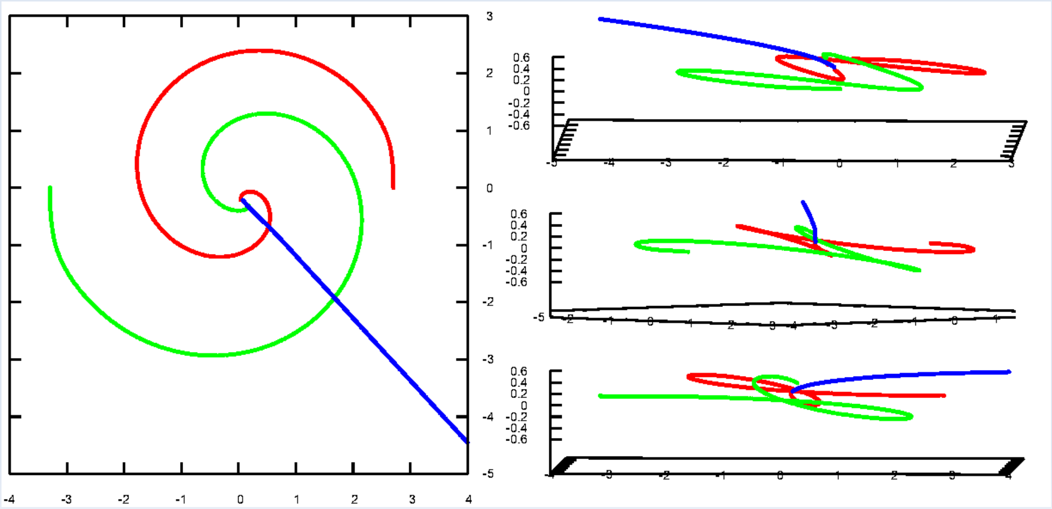
We obtained the values of recoil speeds to be approximately

200 km/s or 300 km/s, which roughly correspond to the two

pre-merging scenarios (details in Janiuk et al. 2013, A&A, 560, 25)

** More massive black hole had spin 0.8, less massive companion is non-spinning

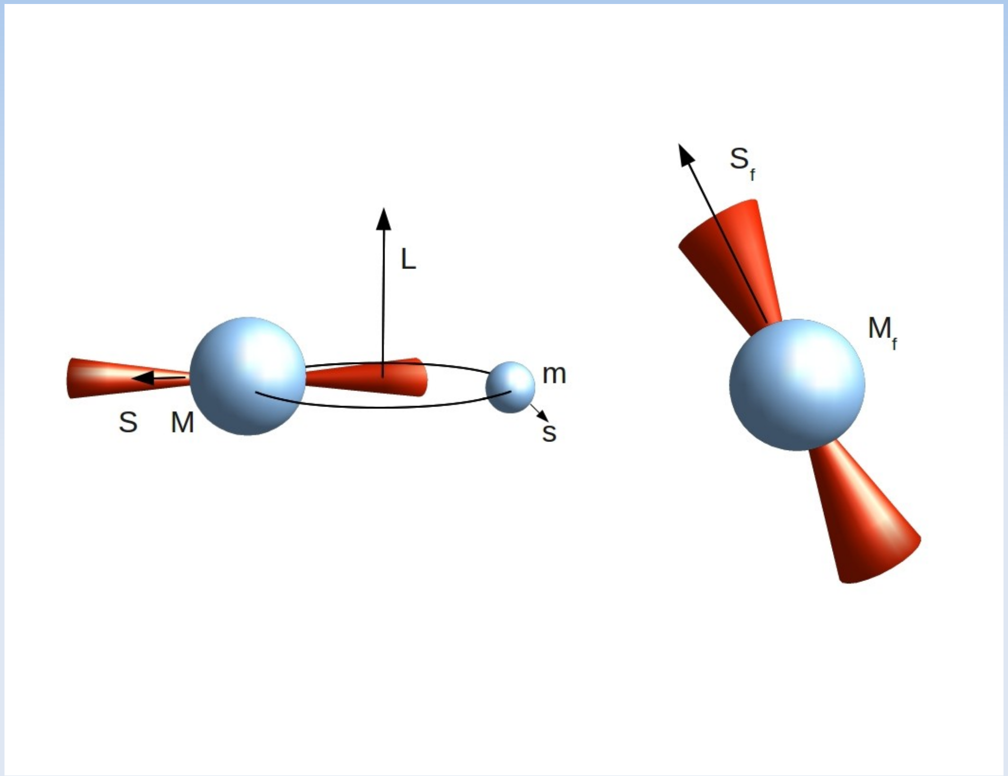
Simulation: S. Charzyński



Spin flip

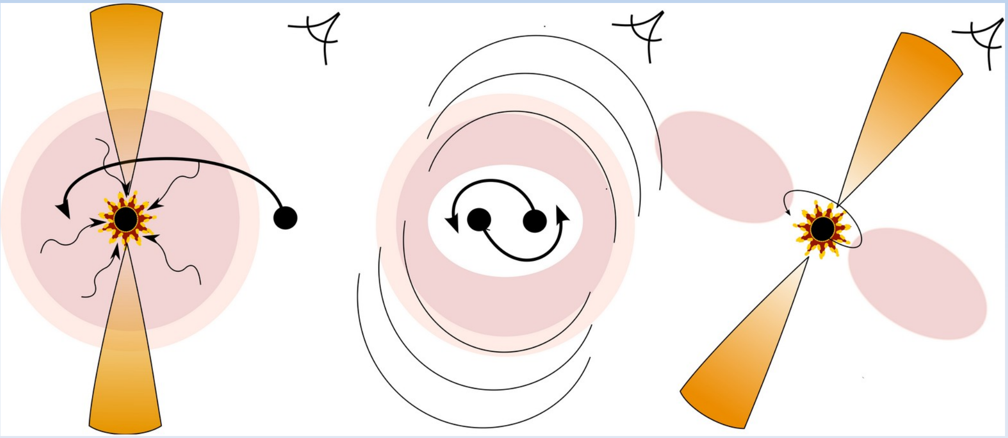
Spin vector of a rotating black hole changes its orientation after the merger

This follows from conservation of total angular momentum, which is the sum of the orbital angular momentum of the system, and spin angular momenta of the two black holes



Can BBH merger produce electromagnetic signal?

- In vacuum, no
- We need matter, like in AGN (”wet merger”)
- The black hole merger inside common envelope, could produce jet, due to accretion: accompanying Gamma Ray Burst!
- We have GW signal from binary black hole merger
- Spin flip may redirect jet axis

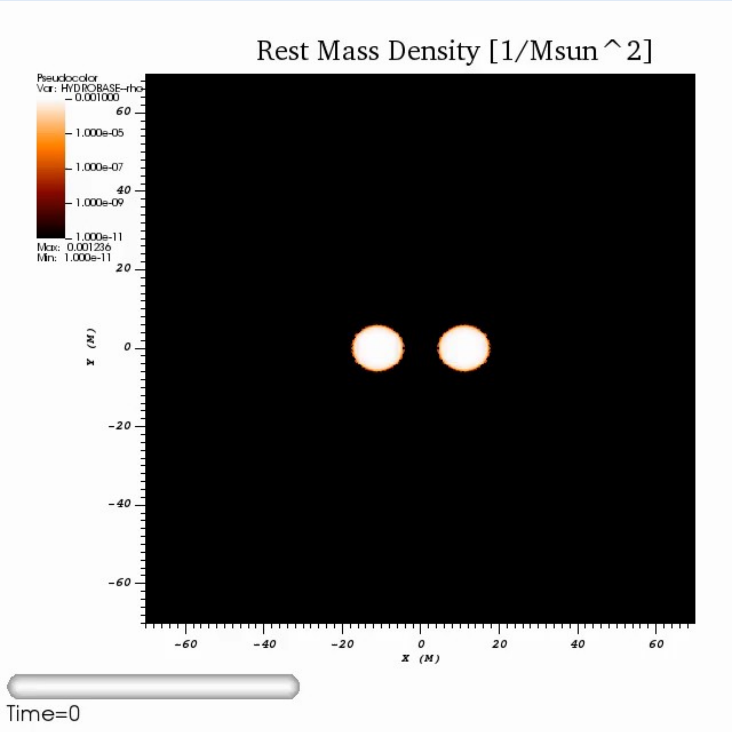


Cartoon scenario for electromagnetic counterpart in GW (Janiuk et al. 2017)

Why study binary black holes

- They are expected to be strongest sources of gravitational waves, detectable by ground-based interferometers
- Hopefully we shall understand better the gravity in strong field limit
- Numerical relativity implemented to solve evolution schemes and overcome difficulties in discretizing field equations $\rightarrow$ progress in computational techniques; community effort
- Simulations are useful to test merger process with astrophysically relevant initial conditions

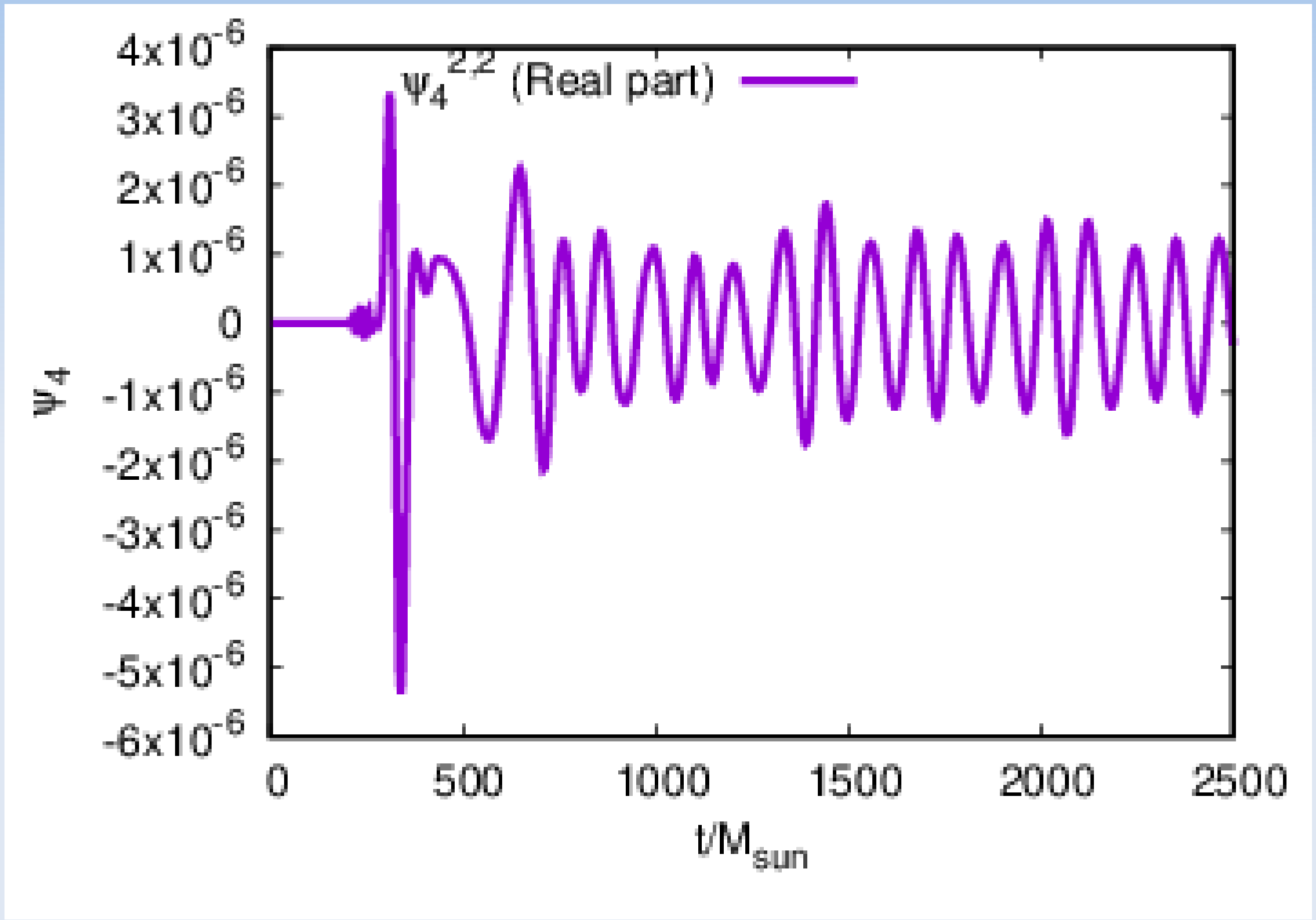
Binary neutron star merger simulation



Density distribution of two merging neutron stars, mass ratio q=1. Evolved with Einstein Toolkit code.

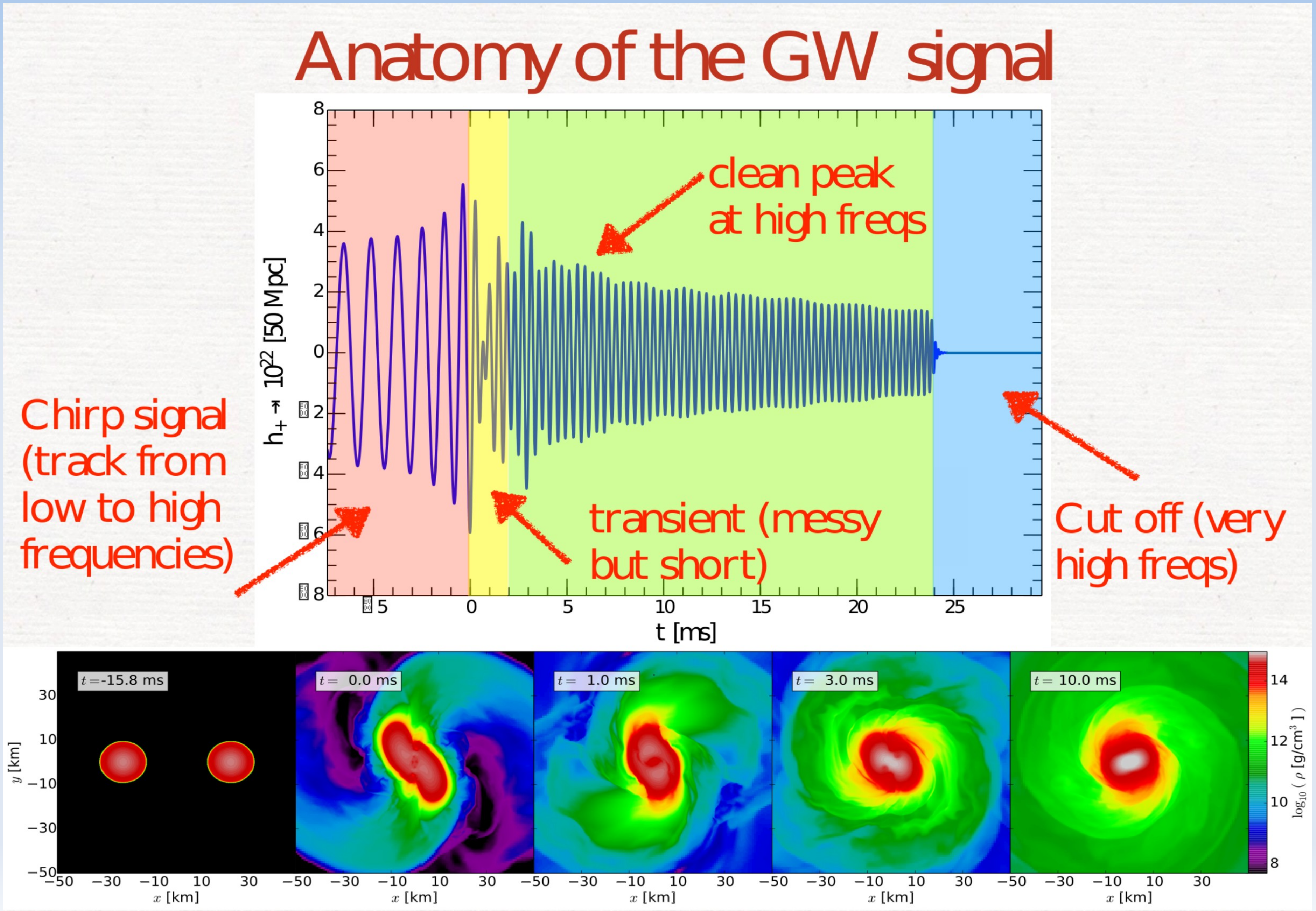
Initial data from: http://www.compact-binaries.org/content/data/bns-initial-data

Gravitational waves from BNS



Plot of 'gravitational wave' (real part of the fluctuating curvature tensor, Weyl 4th component)

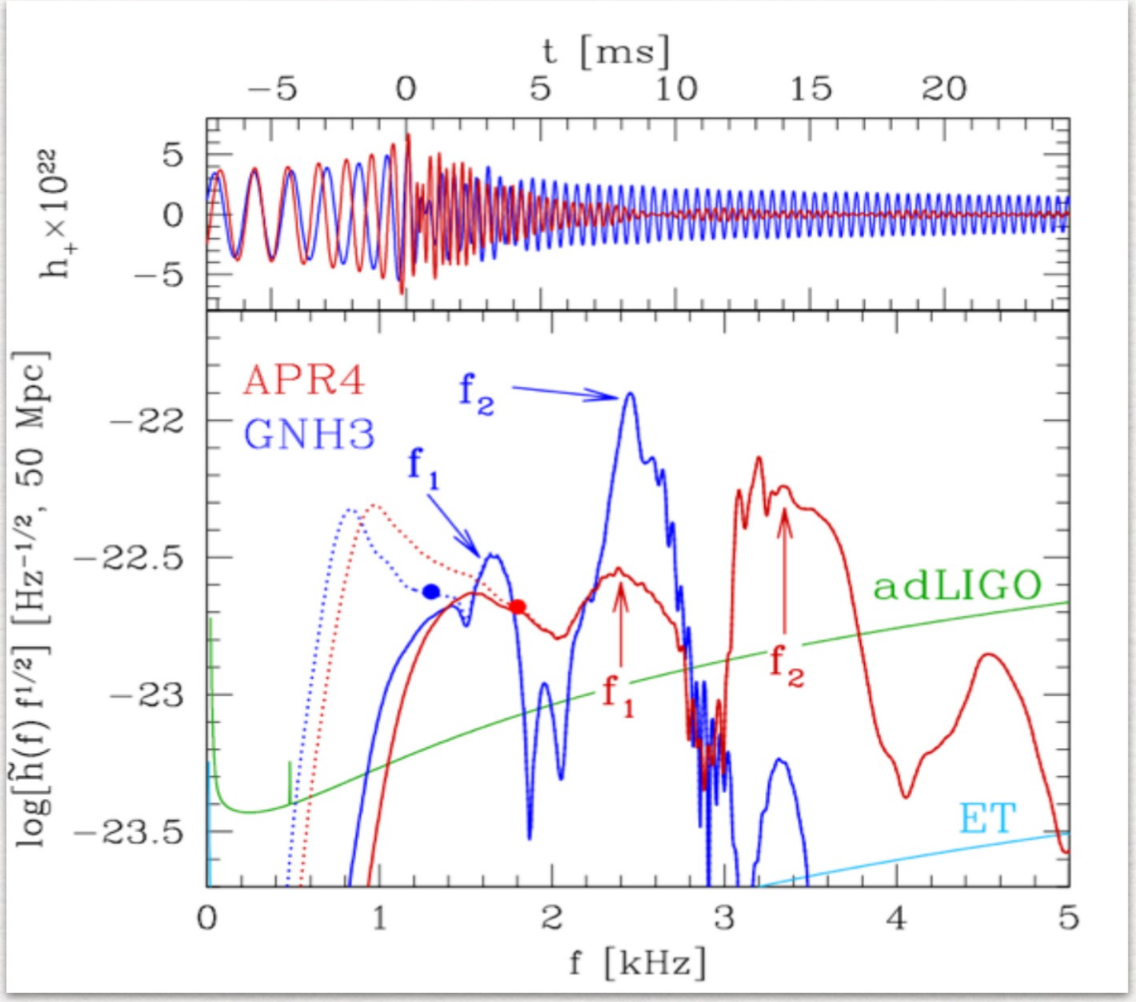
Signal in GWs from binary neutron stars



GW shape and frequency shift

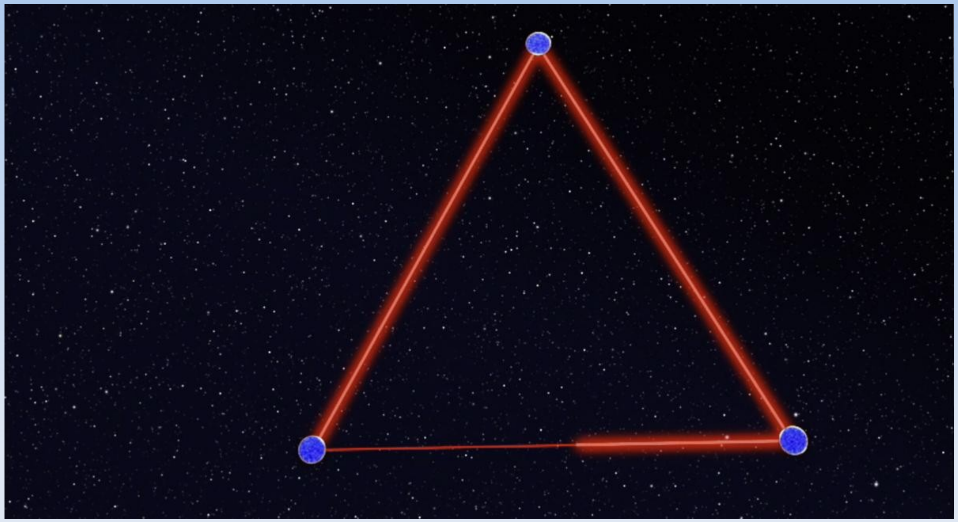
A new approach to constrain the EOS

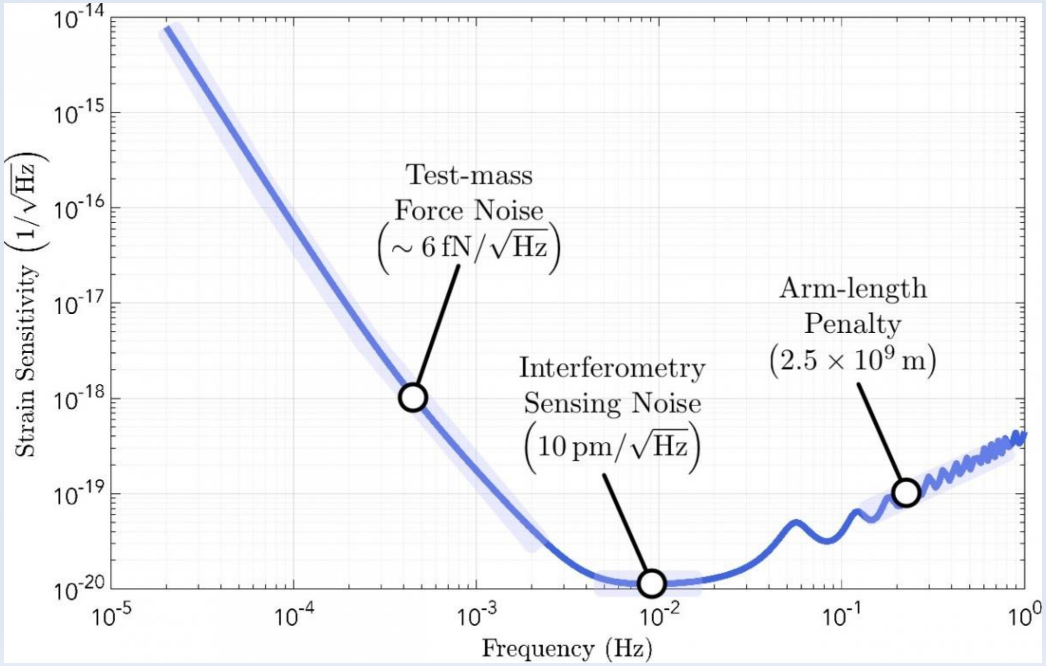
Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013,Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, LR+2016…



Future: Space detector

- Planned space mission for low frequency GWs: LISA
- The three satellites, separated by millions of km, will form a high precision interferometer that senses gravitational waves by monitoring the minute changes in distance between free falling test masses inside the spacecraft.





Next week

- Supernovae

Further reading:

- Aasi et al. ”Characterization of the LIGO detectors during their sixth science run”, 2015, https://arxiv.org/abs/1410.7764
- Lasky, P. ”Gravitational Waves from Neutron Stars: A Review”, 2015, https://arxiv.org/pdf/1508.06643.pdf